

حمل الآن

مجاناً وحصرياً

المراجعة رقم (1)

اختبار شهر نوفمبر





Test 1

1 Choose the correct answer from those given :

(1) If $f(x) = x^3 + 7$, then $f^{-1}(-1) = \dots\dots\dots$

- (a) -1 (b) -2 (c) -3 (d) -4

(2) The function f where $f(x) = a^x$ is decreasing on its domain if $\dots\dots\dots$

- (a) $a = 1$ (b) $a > 1$ (c) $0 < a < 1$ (d) $a = -1$

(3) If $f(x) = \begin{cases} \frac{\tan 3x}{x}, & x > 0 \\ 2 + \cos x, & x \leq 0 \end{cases}$, then $\lim_{x \rightarrow 0} f(x) = \dots\dots\dots$

- (a) 2 (b) 3 (c) 4 (d) $\frac{3}{2}$

(4) $\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x) = \dots\dots\dots$

- (a) $\frac{1}{2}$ (b) zero (c) $\sqrt{2}$ (d) not exist

(5) If $3^{x-2} = 2^{x-2}$, then $x = \dots\dots\dots$

- (a) 3 (b) -2 (c) zero (d) 2

(6) $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^2 + 4} + 5x}{4x + 3} = \dots\dots\dots$

- (a) ∞ (b) 5 (c) 3 (d) 2

(7) If $f(x) = 2^x$, then the value of x which satisfies : $f(x+1) - f(x-1) = 24$ equals $\dots\dots\dots$

- (a) 16 (b) 4 (c) 8 (d) 2

(8) $\lim_{x \rightarrow 0} \frac{\sin 2x + 5 \tan 3x}{x} = \dots\dots\dots$

- (a) 2 (b) 15 (c) 21 (d) 17

(9) The measure of the greatest angle in the triangle whose side lengths are 3 cm., 5 cm., 7 cm. equals $\dots\dots\dots$

- (a) 150° (b) 120° (c) 60° (d) 30°

(10) The solution set of the inequality : $|2x - 6| + |3 - x| > 12$ is $\dots\dots\dots$

- (a) $]-1, 7[$ (b) $\mathbb{R} - [-3, 9]$ (c) $\mathbb{R} - [-1, 7]$ (d) $]-3, 9[$

(11) In ΔXYZ if $X = y$, then $\cos X = \dots\dots\dots$

(a) $\frac{2y^2}{z}$

(b) $\frac{z}{2y}$

(c) $\frac{z}{4x}$

(d) $\frac{y}{2x}$

(12) The domain of the function $f : f(x) = \log_{(1-x)} 3$ is $\dots\dots\dots$

(a) $]-\infty, 1[$

(b) $]1, \infty[$

(c) $]-1, 1[$

(d) $]-\infty, 0[\cup]0, 1[$

2 Answer the following two questions :

(1) Find : $\lim_{x \rightarrow 0} \cot 3x \sin 5x$

(2) Find algebraically in $\mathbb{R}$ the solution set of the inequality : $\sqrt{(x+2)^2} + |2x+4| \leq 6$

1 Choose the correct answer from those given :

(1) If $f(x) = \begin{cases} 3x^2 + ax - 2, & x > 3 \\ 2x + b, & x < 3 \end{cases}$ and if $\lim_{x \rightarrow 3} f(x) = 16$, then $a + b = \dots\dots\dots$

- (a) 4 (b) 7 (c) -1 (d) 13

(2) The solution set of the inequality $\sqrt{x^2 - 4x + 4} > 0$ in $\mathbb{R}$ is $\dots\dots\dots$

- (a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{-2\}$ (c) $\mathbb{R}$ (d) $\emptyset$

(3) $\lim_{x \rightarrow \infty} \frac{2x + 3}{5x^2 + 4} = \dots\dots\dots$

- (a) 2 (b) zero (c) $\frac{3}{4}$ (d) $\frac{2}{5}$

(4) If the point $(k, 3)$ lies on the curve of the function $f : f(x) = \log_3(x + 7)$, then $k = \dots\dots\dots$

- (a) 3 (b) 27 (c) 9 (d) 20

(5) If $f(x) = 2^x$, then $\frac{f(x+1) + f(x)}{f(x-1)} = \dots\dots\dots$

- (a) 3 (b) 6 (c) 4 (d) 8

(6) If $(2x - 1)^4 = 81$, then $x \in \dots\dots\dots$

- (a) $\{1\}$ (b) $\{1, -2\}$ (c) $\{2\}$ (d) $\{-1, 2\}$

(7) In any triangle XYZ, $x^2 + y^2 - 2xy \sin(90^\circ - Z) = \dots\dots\dots$

- (a) x^2 (b) y^2 (c) z^2 (d) z

(8) $\lim_{x \rightarrow 0} \frac{2x \cos 8x + 2 \sin 5x}{\tan 2x} = \dots\dots\dots$

- (a) 13 (b) 10 (c) 9 (d) 6

(9) If the function $f : f(x) = 5x + b$ is the inverse function of the function

$g : g(x) = cx + 4$, then $b \times c = \dots\dots\dots$

- (a) 4 (b) -4 (c) 20 (d) -20

(10) The solution set of the equation : $3^{x+1} + 3^x = 36$ in $\mathbb{R}$ is

- (a) $\{0\}$ (b) $\{1\}$ (c) $\{2\}$ (d) $\{0, 2\}$

(11) $\lim_{x \rightarrow \infty} \sqrt{\frac{3+2x}{4x-1}} = \dots\dots\dots$

- (a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

(12) In the opposite figure :

ABCD is a quadrilateral in which $AB = 8$ cm.

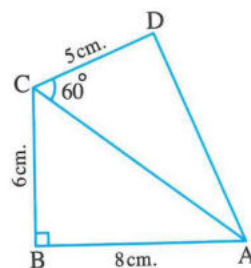
, $BC = 6$ cm. , $m(\angle B) = 90^\circ$

, $DC = 5$ cm. and $m(\angle ACD) = 60^\circ$

, then the area of the circumcircle of

the triangle ADC = cm^2

- (a) 9π (b) 16π (c) 25π (d) 49π

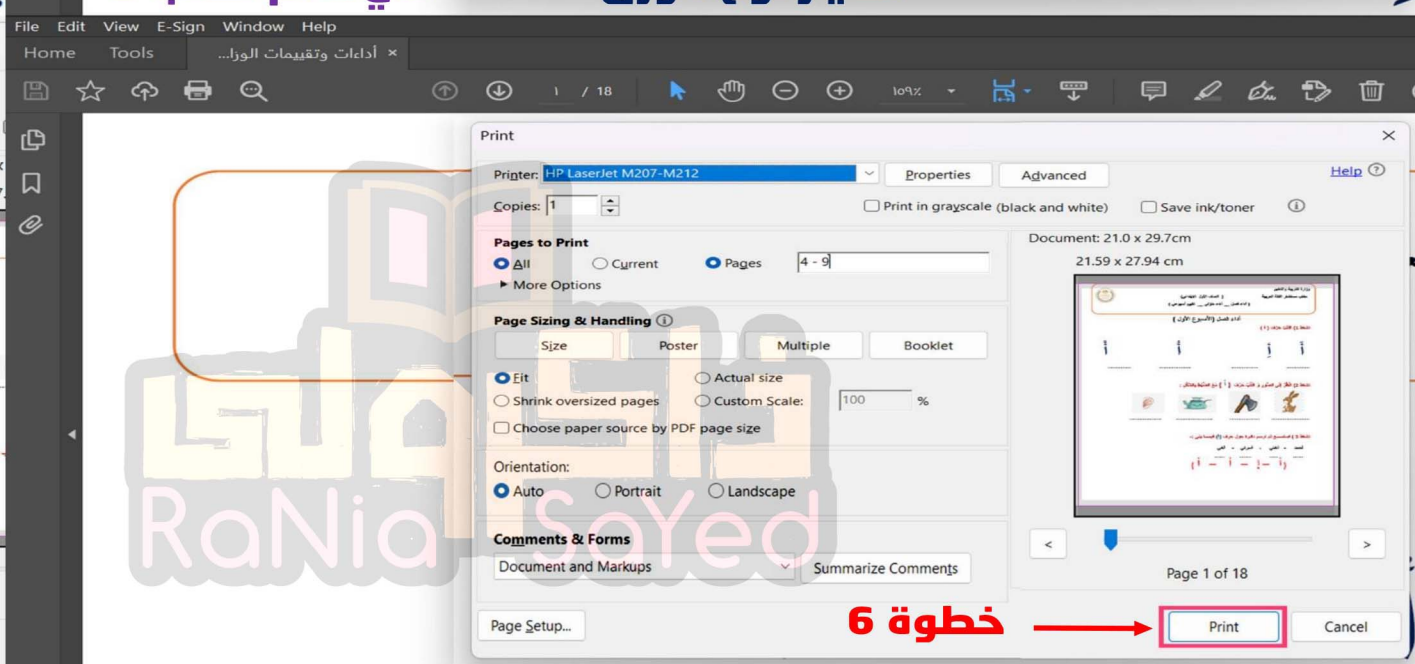
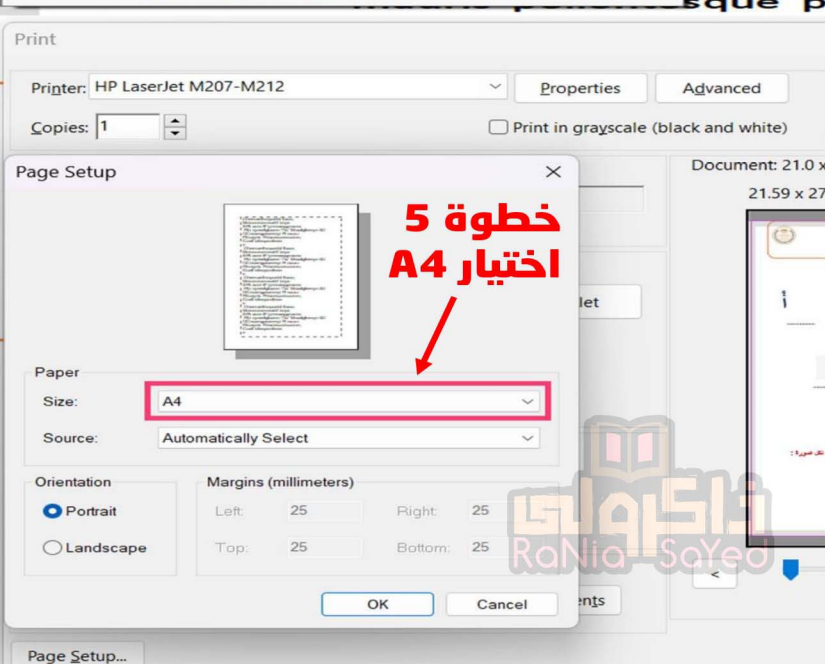
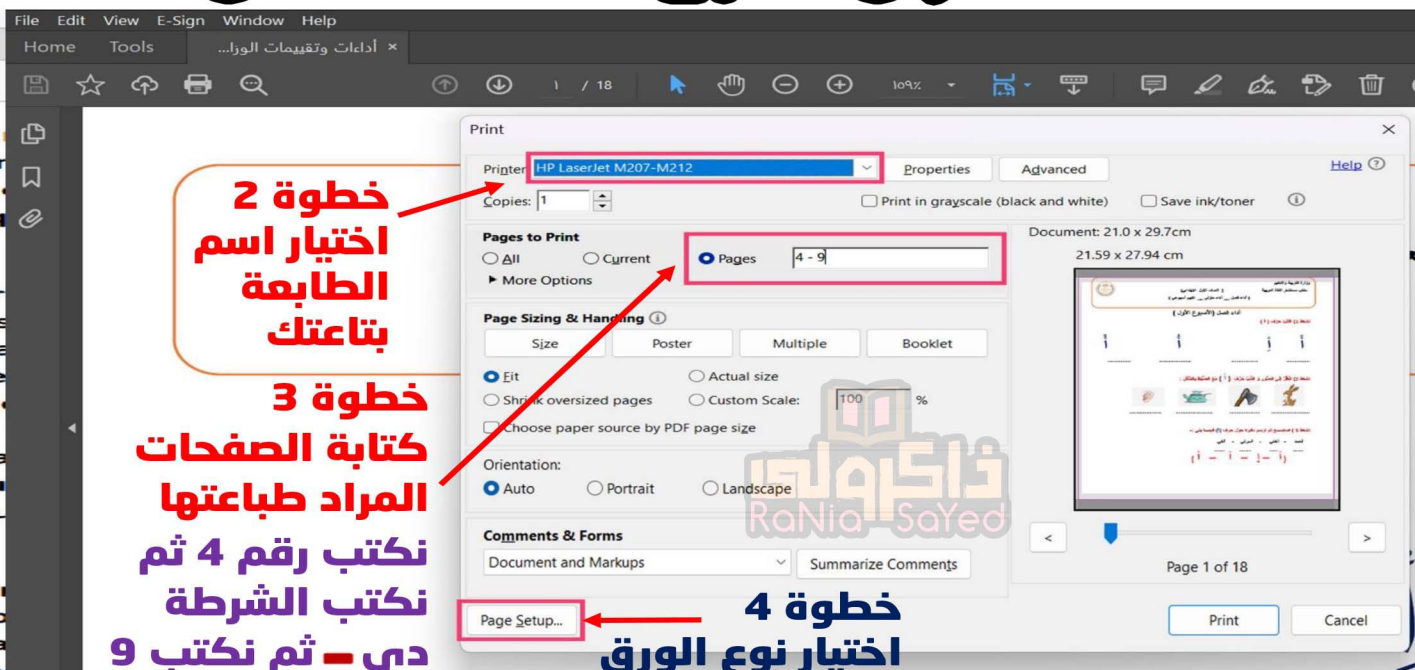
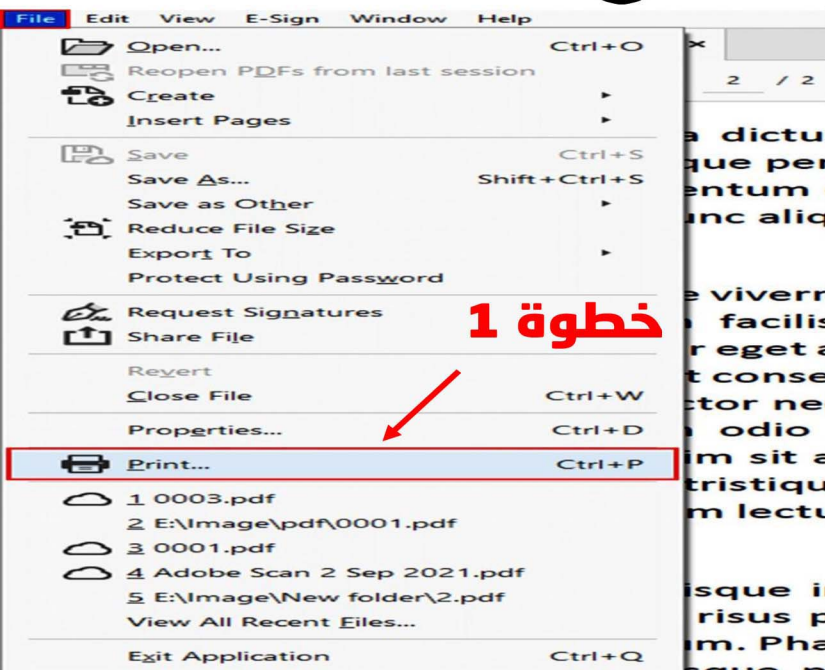


2 Answer the following two questions :

(1) If $f(x) = 5^x$, find the solution set in $\mathbb{R}$ of the equation : $f(x) + f(x-1) = 150$

(2) Find : $\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{x+1}-1}$

كيفية طباعة صفحات معينة من ملف معين مثلا ازاي نطبع الصفحات من صفحة 4 الى صفحة 9



حمل الآن

مجاناً وحصرياً

المراجعة رقم (2)

اختبار شهر نوفمبر



**Lesson (5)****Equations and Inequalities****Lesson objectives****Related Links**

-  Solve the modulus equations graphically.
-  Solve the modulus equations algebraically.
-  Solve the modulus inequalities graphically.
-  Solve the modulus in equalities algebraically.

**Definition:****The absolute value of a real number:**

If x is a real number, then the symbol $|x|$ is called the absolute value of the number x and it is define as follows: $|x| = \begin{cases} x & \text{at } x \geq 0 \\ -x & \text{at } x < 0 \end{cases}$

Remarks:

- ① For any real number a we get: $|a| = |-a|$
- ② $|x - a| = |a - x|$
- ③ $|x| = c > 0 \Leftrightarrow x = c \text{ or } x = -c$
- ④ If a, b are two real numbers, then $|a| = |b| \Leftrightarrow a = b \text{ or } a = -b$
- ⑤ For any real number a it will be: $(|a|)^2 = a^2$

- | | |
|---|-----------------------------------|
| ① $ x < a \Leftrightarrow -a < x < a$ | i.e. $x \in]-a, a[$ |
| ② $ x \leq a \Leftrightarrow -a \leq x \leq a$ | i.e. $x \in [-a, a]$ |
| ③ $ x - b < a \Leftrightarrow b - a < x < b + a$ | i.e. $x \in]b - a, b + a[$ |
| ④ $ x - b \leq a \Leftrightarrow b - a \leq x \leq b + a$ | i.e. $x \in [b - a, b + a]$ |
| ⑤ $ x > a \Leftrightarrow x > a \text{ or } x < -a$ | i.e. $x \in \mathbb{R} - [-a, a]$ |
| ⑥ $ x \geq a \Leftrightarrow x \geq a \text{ or } x \leq -a$ | i.e. $x \in \mathbb{R} -]-a, a[$ |



Example (1)

Find algebraically the solution set of each of the following equation in $\mathbb{R}$:

(1) $|x| - 4 = 0$

Solution: $\because |x| - 4 = 0$

$\therefore x = 4 \quad \text{or} \quad x = -4$

$\therefore |x| = 4$

$\therefore \text{The S.S.} = \{-4, 4\}$

(2) $|2x + 3| - 2 = 0$

Solution: $\because |2x + 3| - 2 = 0$

$\therefore 2x + 3 = 2$

$\therefore 2x = 2 - 3$

$\therefore 2x = -1$

$\therefore x = -\frac{1}{2}$

$\therefore \text{The S.S.} = \{-\frac{1}{2}, -\frac{5}{2}\}$

or

or

or

or

$\therefore |2x + 3| = 2$

$2x + 3 = -2$

$2x = -2 - 3$

$2x = -5$

$x = -\frac{5}{2}$

(3) $|x - 3| + 2 = 0$

Solution: $\because |x - 3| + 2 = 0$

$\therefore |x - 3| = -2 \quad (\text{refused})$

$\therefore \text{The S.S.} = \emptyset$

(4) $6 - 2|x + 1| = 0$

Solution: $\because 6 - 2|x + 1| = 0$

$\therefore |x + 1| = 3$

$\therefore x + 1 = 3$

$\therefore x = 3 - 1$

$\therefore x = 2$

$\therefore \text{The S.S.} = \{2, -4\}$

or

or

or

$\therefore 2|x + 1| = 6$

$x + 1 = -3$

$x = -3 - 1$

$x = -4$



Example (2)

Find algebraically the solution set of each of the following equation in $\mathbb{R}$:

(1) $|x + 2| + x = 0$

Solution: $\because |x + 2| + x = 0$

$x \geq -2$

$\therefore x + 2 + x = 0$

$\therefore 2x = -2$

$\therefore x = -1 \quad (\checkmark)$

$\therefore \text{The S.S.} = \{-1\}$

or

$x < -2$

~~$-x - 2 + x = 0$~~ **(refused)**

(2) $|2x + 4| = x - 3$

Solution: $\because |2x + 4| = x - 3$

$x \geq -2$

$\therefore 2x + 4 = x - 3$

$\therefore 2x - x = -3 - 4$

$\therefore x = -7 \quad (\times)$

or

$x < -2$

$-2x - 4 = x - 3$

$-2x - x = -3 + 4$

$-3x = 1$

$x = -\frac{1}{3} \quad (\times)$

$\therefore \text{The S.S.} = \emptyset$

(3) $x^2 - 3|x - 2| - 4 = 0$

Solution: $\because x^2 - 3|x - 2| - 4 = 0$

$x \geq 2$

$\therefore x^2 - 3(x - 2) - 4 = 0$ or

$\therefore x^2 - 3x + 6 - 4 = 0$ or

$\therefore x^2 - 3x + 2 = 0$ or

MODE $\rightarrow$ **EQN** $\rightarrow$ **3**

$\therefore x = 2 \quad (\checkmark)$

& $x = 1 \quad (\times)$

$\therefore \text{The S.S.} = \{2, -5\}$

or

$x < 2$

$x^2 - 3(-x + 2) - 4 = 0$

$x^2 + 3x - 6 - 4 = 0$

$x^2 + 3x - 10 = 0$

$x = 2 \quad (\times)$

$x = -5 \quad (\checkmark)$



(4) $x |x| + 3x - |x| = 3$

Solution: $\because x |x| + 3x - |x| = 3$

$x \geq 0$

$\therefore x(x) + 3x - x = 3$

$\therefore x^2 + 2x - 3 = 0$

$\therefore x = 1 \quad (\checkmark)$

& $x = -3 \quad (\times)$

$\therefore$ The S.S. = $\{1\}$

$x < 0$

$x(-x) + 3x + x = 3$

$-x^2 + 4x - 3 = 0$

$x = 3 \quad (\times)$

$x = 1 \quad (\times)$

(5) $|x| (3 + |x|) - 4 = 0$

Solution: $\because |x| (3 + |x|) - 4 = 0$

$x \geq 0$

$\therefore x(3 + x) - 4 = 0$

$\therefore 3x + x^2 - 4 = 0$

$\therefore x^2 + 3x - 4 = 0$

MODE $\rightarrow$ EQN $\rightarrow$ 3

$\therefore x = 1 \quad (\checkmark)$

& $x = -4 \quad (\times)$

$\therefore$ The S.S. = $\{1, -1\}$

$x < 0$

$-x(3 - x) - 4 = 0$

$-3x + x^2 - 4 = 0$

$x^2 - 3x - 4 = 0$

$x = 4 \quad (\times)$

$x = -1 \quad (\checkmark)$

(6) $\sqrt{x^2 + 2x + 1} = 5$

Solution: $\because \sqrt{x^2 + 2x + 1} = 5$

$\therefore \sqrt{(x + 1)^2} = 5$

$\therefore x + 1 = 5$

$\therefore x = 4$

$\therefore$ The S.S. = $\{4, -6\}$

$\therefore |x + 1| = 5$

$x + 1 = -5$

$x = -6$



(7) $\sqrt{x^2 - 6x + 9} + 2x = 9$

Solution: $\therefore \sqrt{x^2 - 6x + 9} + 2x = 9$

$\therefore \sqrt{(x-3)^2} + 2x = 9$

$x \geq 3$

$\therefore x - 3 + 2x = 9$

$\therefore 3x - 3 = 9$

$\therefore 3x = 12$

$\therefore x = 4 \quad (\checkmark)$

$\therefore \text{The S.S.} = \{4\}$

$\therefore |x - 3| + 2x = 9$

$x < 3$

$-x + 3 + 2x = 9$

$x + 3 = 9$

$x = 6 \quad (\times)$

Example (3)

Find algebraically the solution set of each of the following equation in $\mathbb{R}$:

(1) $|x + 5| = |x - 3|$

Solution: $\therefore |x + 5| = |x - 3|$

	-5	3	
$-5 > x$	$-5 \leq x < 3$	$x \geq 3$	
$\cancel{x} - 5 = \cancel{x} + 3$ <i>refused</i>	$x + 5 = -x + 3$ $\therefore x + x = 3 - 5$ $\therefore 2x = -2$ $\therefore x = -1 \quad (\checkmark)$	$\cancel{x} + 5 = \cancel{x} - 3$ <i>refused</i>	

$\therefore \text{The S.S.} = \{-1\}$

(2) $|x - 2| = |x| - 1$

Solution: $\therefore |x - 2| = |x| - 1$

	0	2	
$0 > x$	$0 \leq x < 2$	$x \geq 2$	
$\cancel{x} + 2 = \cancel{x} - 1$ <i>refused</i>	$-x + 2 = x - 1$ $\therefore -x - x = -1 - 2$ $\therefore -2x = -3$ $\therefore x = \frac{3}{2} \quad (\checkmark)$	$\cancel{x} - 2 = \cancel{x} - 1$ <i>refused</i>	

$\therefore \text{The S.S.} = \{\frac{3}{2}\}$

**PRACTICE (1)**

Q1: What is the solution set of the equation $|x| = 94$?

- ☐ A $\{-94, 94\}$
- ☐ B $\{-94\}$
- ☐ C $\{-95, 95\}$
- ☐ D $\{94\}$

Q2: What is the solution set of the equation $3|x| - 66 = 0$?

- ☐ A $\{-22\}$
- ☐ B $\{-22, 22\}$
- ☐ C $\{22\}$
- ☐ D $\{-66, 66\}$

Q3: Find the solution set of the equation $|x + 3| = -3x + 7$.

- ☐ A $\{5\}$
- ☐ B $[1, 5]$
- ☐ C $\{1\}$
- ☐ D $[-5, -1]$
- ☐ E $\{1, 5\}$

Q4: Write the set of all solutions to the equation $|x + 6| = 3$.

- ☐ A $\{-9, 9\}$
- ☐ B $\{3, 9\}$
- ☐ C $\{9\}$
- ☐ D $\{3, -3\}$
- ☐ E $\{-9, -3\}$



Q5: Find the solution set of $|x + 3| = |2x - 6|$.

- ☐ A $\{-9\}$
- ☐ B $\{-1\}$
- ☐ C $\{9, 1\}$
- ☐ D $\{1\}$
- ☐ E $\{9\}$

Q6: Find algebraically the solution set of the equation $|x + 4| = x + 4$.

- ☐ A $] -\infty, -4]$
- ☐ B $[-4, \infty[$
- ☐ C $[4, \infty[$
- ☐ D $] -\infty, 4]$

Q7: Find algebraically the solution set of the equation $4x|x| - 4x = 0$.

- ☐ A $\{-4, 0, 4\}$
- ☐ B $\{-1, 1\}$
- ☐ C $\{0, 1\}$
- ☐ D $\{-1, 0, 1\}$

Q8: Find algebraically the solution set of the equation $|x + 3||x - 3| = 39$.

- ☐ A $\{4\sqrt{3}\}$
- ☐ B $\{-\sqrt{30}, \sqrt{30}\}$
- ☐ C $\{-4\sqrt{3}, 4\sqrt{3}\}$
- ☐ D $\{\sqrt{30}, 4\sqrt{3}\}$



SOLVING INEQUALITIES ALGEBRICALLY

Example (4)

Find algebraically the solution set of each of the following inequalities in $\mathbb{R}$:

(1) $\left| \frac{x}{3} \right| \leq 1$

Solution: $\because \left| \frac{x}{3} \right| \leq 1$

$$\therefore -1 \leq \frac{x}{3} \leq 1$$

$$\therefore -1 \times 3 \leq x \leq 1 \times 3$$

$$\therefore -3 \leq x \leq 3$$

$$\therefore \text{The S.S.} = [-3, 3]$$

(2) $|2x - 5| < 1$

Solution: $\because |2x - 5| < 1$

$$\therefore -1 < 2x - 5 < 1$$

$$\therefore -1 + 5 < 2x < 1 + 5$$

$$\therefore 4 < 2x < 6$$

$$\therefore 2 < x < 3$$

$$\therefore \text{The S.S.} =]2, 3[$$

(3) $|x + 2| > 3$

Solution: $\because |x + 2| > 3$

$$\therefore x + 2 > 3$$

or

$$x + 2 < -3$$

$$\therefore x > 3 - 2$$

or

$$x < -3 - 2$$

$$\therefore x > 1$$

or

$$x < -5$$

$$\therefore \text{The S.S.} = \mathbb{R} - [-5, 1]$$

(4) $\left| \frac{x}{2} \right| > 1$

Solution: $\because \left| \frac{x}{2} \right| > 1$

$$\therefore \frac{x}{2} > 1$$

or

$$\frac{x}{2} < -1$$

$$\therefore x > 2$$

or

$$x < -2$$

$$\therefore \text{The S.S.} = \mathbb{R} - [-2, 2]$$



(5) $|3x - 1| \geq 5$

Solution: $\because |3x - 1| \geq 5$

$$\therefore 3x - 1 \geq 5 \quad \text{or} \quad 3x - 1 \leq -5$$

$$\therefore 3x \geq 5 + 1 \quad \text{or} \quad 3x \leq -5 + 1$$

$$\therefore 3x \geq 6 \quad \text{or} \quad 3x \leq -4$$

$$\therefore x \geq 2 \quad \text{or} \quad x \leq \frac{-4}{3}$$

$$\therefore \text{The S.S.} = \mathbb{R} -] -\frac{4}{3}, 2 [$$

(6) $\frac{\sqrt{x^2 + 4x + 4}}{3} > 1$

Solution: $\because \frac{\sqrt{x^2 + 4x + 4}}{3} > 1$

$$\therefore \sqrt{x^2 + 4x + 4} > 3 \quad \therefore \sqrt{(x + 2)^2} > 3$$

$$\therefore |x + 2| > 3$$

$$\therefore x + 2 > 3 \quad \text{or} \quad x + 2 < -3$$

$$\therefore x > 3 - 2 \quad \text{or} \quad x < -3 - 2$$

$$\therefore x > 1 \quad \text{or} \quad x < -5$$

$$\therefore \text{The S.S.} = \mathbb{R} - [-5, 1]$$

(7) $|3x - 1| \geq 5$

Solution: $\because |3x - 1| \geq 5$

$$\therefore 3x - 1 \geq 5 \quad \text{or} \quad 3x - 1 \leq -5$$

$$\therefore 3x \geq 5 + 1 \quad \text{or} \quad 3x \leq -5 + 1$$

$$\therefore 3x \geq 6 \quad \text{or} \quad 3x \leq -4$$

$$\therefore x \geq 2 \quad \text{or} \quad x \leq \frac{-4}{3}$$

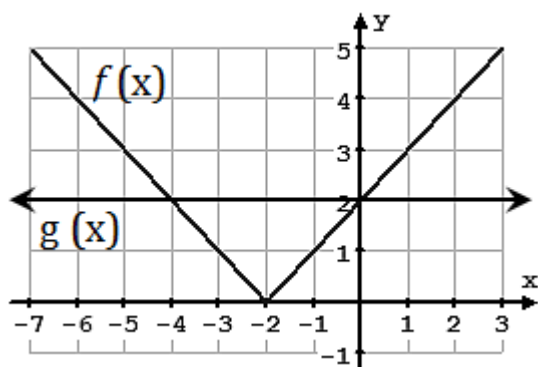
$$\therefore \text{The S.S.} = \mathbb{R} -] -\frac{4}{3}, 2 [$$



SOLVING INEQUALITIES GRAPHICALLY

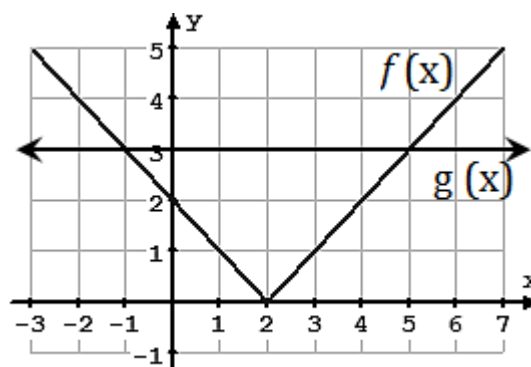
Example (5)

Find graphically the solution set of each of the following inequalities in $\mathbb{R}$:



$$|x + 2| < 2$$

$$\text{S.S.} =]0, 4[$$

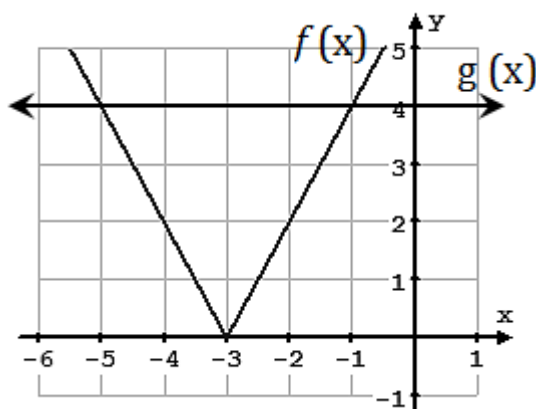


$$|x - 2| \leq 3$$

$$\text{S.S.} = [-1, 5]$$

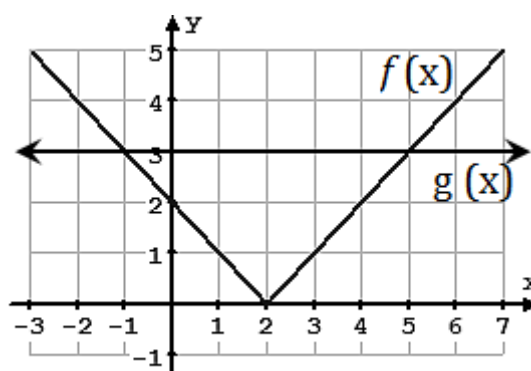
Example (6)

Find graphically the solution set of each of the following inequalities in $\mathbb{R}$:



$$|2x + 4| \geq 4$$

$$\text{S.S.} = \mathbb{R} -]-5, -1[$$



$$|x - 2| > 3$$

$$\text{S.S.} = \mathbb{R} - [-1, 5]$$

**PRACTICE (2)**

Q1: Find the solution set of the inequality $|x + 4| < 9$.

- ☐ A $\{-13, 5\}$
- ☐ B $] -13, 5[$
- ☐ C $] -5, 13[$
- ☐ D $] 13, \infty[$
- ☐ E $] -\infty, 5[$

Q2: Find algebraically the solution set of the inequality $|7 - x| + 3 \leq -6$.

- ☐ A $] -2, 16[$
- ☐ B $\emptyset$
- ☐ C $[16, \infty[$
- ☐ D $\mathbb{R} - [-16, 2]$

Q3: Find algebraically the solution set of the inequality $|6 - x| < 3$.

- ☐ A $] -9, -3[$
- ☐ B $] 3, \infty[$
- ☐ C $\mathbb{R} - [3, 9]$
- ☐ D $] 3, 9[$

Q4: Find the solution set of the inequality $|x - 6| \geq 7$.

- ☐ A $\mathbb{R} - [-13, 1]$
- ☐ B $\mathbb{R} - \{-1, 13\}$
- ☐ C $[1, \infty[$
- ☐ D $[13, \infty[$
- ☐ E $\mathbb{R} -] -1, 13[$



Q5: Find algebraically the solution set of the inequality $|8 - x| > 17$.

- ☐ A $] -9, 25[$
- ☐ B $\mathbb{R} - [-9, 25]$
- ☐ C $\mathbb{R} - [-25, 9]$
- ☐ D $] -\infty, -9[$

Q6: A factory produces cans with weight x grams. To control the production quality, the cans are only allowed to be sold if $|x - 183| \leq 6$. Determine the heaviest and the lightest weight of a can that can be sold.

- ☐ A 189 g, 183 g
- ☐ B 183 g, 6 g
- ☐ C 183 g, 177 g
- ☐ D 189 g, 177 g

Q7: Find the solution set of the inequality $|x + 5| > -7$.

- ☐ A $] -12, \infty[$
- ☐ B $\mathbb{R} - \{-12, 2\}$
- ☐ C $\mathbb{R} - [-2, 12]$
- ☐ D $] -2, \infty[$
- ☐ E $\mathbb{R}$

Q8: Find the solution set of the inequality $|x - 3| \leq 7$.

- ☐ A $] -\infty, 10]$
- ☐ B $[4, \infty[$
- ☐ C $[-10, 4]$
- ☐ D $\{-4, 10\}$
- ☐ E $[-4, 10]$

**Exercise (1 - 5)**

Find the solution set of each of the following equations algebraically:

① $|x - 2| = 3$

② $|3 - 2x| = 7$

③ $|x + 2| = 3x - 10$

④ $|x + 2| + x - 2 = 0$

⑤ $x + |x| = 2$

⑥ $|x - 2| = 3x - 4$

⑦ $|x - 1| = x - 2$

⑧ $|2x - 6| = |x - 3|$

⑨ $\sqrt{x^2 - 6x + 9} + 2x = 9$

Find the solution set of each of the following equations graphically:

⑩ $|x - 3| = 7$

⑪ $|x + 2| + x - 2 = 0$

⑫ $|x - 2| = 3x - 4$

⑬ $|2x - 4| = |x + 1|$

⑭ $|x| + x = 0$

⑮ $|x + 2| = |x - 3|$

Find the solution set of each of the following inequalities graphically:

⑯ $|x - 1| < 2$

⑰ $|x - 2| < 3$

⑱ $|5 - x| > 3$

⑲ $|2x - 3| \geq 7$

⑳ $|x + 3| > -1$

㉑ $|2x - 5| \geq 2$

Find the solution set of each of the following inequalities algebraically:

㉒ $|x - 3| \leq 15$

㉓ $|3x - 2| < 4$

㉔ $|3x - 7| \geq 2$

㉕ $|3x + 2| + 5 < 4$

㉖ $\sqrt{x^2 - 2x + 1} \geq 4$

㉗ $\sqrt{4x^2 - 12x + 9} \leq 9$

㉘ $|2x - 3| + |6 - 4x| < 12$

㉙ $\frac{1}{|2x - 5|} \geq 3$

㉚ $\frac{1}{|2x - 3|} > 2$

**Lesson (1)****Integer Exponential****Lesson objectives****Related Links**

- ✚ Recognize generalization of the laws of the exponents.
- ✚ Determine the n th root.
- ✚ Determine the laws of the rational exponents.

Laws of integer exponents

☐ **For every $m, n \in \mathbb{Z}^+$ we have:**

- ① $a^m \times a^n = a^{m+n}$, $a \in \mathbb{R}$
- ② $\frac{a^m}{a^n} = a^{m-n}$, $a \in \mathbb{R}$
- ③ $(a^m)^n = a^{mn}$, $a \in \mathbb{R}$
- ④ $(a \cdot b)^n = a^n \cdot b^n$, $a, b \in \mathbb{R}$
- ⑤ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $a \in \mathbb{R}, b \in \mathbb{R} - \{0\}$

☐ **For every $m, n \in \mathbb{Z}$ we have:**

- ① If: $a^n = 1$, then $n = \text{zero}$
- ② If: $a^m = a^n$, then $n = m$ for every $a \in \mathbb{R} - \{-1, 1\}$
- ③ If: $a^n = b^n$, then $a = b$ when n is odd and $a = \pm b$ if n is even,
If: $a \neq b$, then $n = \text{zero}$

**Remember that:**

2 ^{....}	4 , 8 , 16 , 32 , 64 , 128 , 256 , 512 , 1024
3 ^{....}	9 , 27 , 81 , 243 , 729
5 ^{....}	25 , 125 , 625
7 ^{....}	49 , 343

Examples**Example (1)****Simplify to the simplest form:**

$$(1) \quad \frac{(32)^3 \times (27)^4}{(72)^5}$$

$$\text{Solution: } \frac{(32)^3 \times (27)^4}{(72)^5} = \frac{(2^5)^3 \times (3^3)^4}{(8 \times 9)^5} = \frac{(2^5)^3 \times (3^3)^4}{(2^3 \times 3^2)^5} = \frac{2^{15} \times 3^{12}}{2^{15} \times 3^{10}} = 3^2 = 9$$

$$(2) \quad \frac{8^{n-1} \times 32^{2-n}}{32 \times 4^{-n}}$$

$$\begin{aligned} \text{Solution: } \frac{8^{n-1} \times 32^{2-n}}{32 \times 4^{-n}} &= \frac{(2^3)^{n-1} \times (2^5)^{2-n}}{2^5 \times (2^2)^{-n}} = \frac{2^{3n-3} \times 2^{10-5n}}{2^5 \times 2^{-2n}} \\ &= 2^{3n-3+10-5n-5+2n} = 2^2 = 4 \end{aligned}$$

$$(3) \quad \frac{6 \times 2^{3-x}}{2^{3-x} \times 2^{1-x}}$$

$$\text{Solution: } \frac{6 \times 2^{3-x}}{2^{3-x} \times 2^{1-x}} = \frac{2 \times 3 \times 2^{3-x}}{2^{3-x} \times 2^{1-x}} = 2^{1+3-x-3+x} \times 3 = 2 \times 3 = 6$$



Example (2)

Prove that:

$$\textcircled{1} \frac{9^{4n+1} \times 4^{2-2n}}{3^{9n+1} \times 48^{1-n}} = 1$$

Prrof

$$\begin{aligned} \text{L.H.S.} &= \frac{9^{4n+1} \times 4^{2-2n}}{3^{9n+1} \times 48^{1-n}} = \frac{(3^2)^{4n+1} \times (2^2)^{2-2n}}{3^{9n+1} \times (3 \times 16)^{1-n}} = \frac{(3)^{8n+2} \times (2)^{4-4n}}{(3)^{9n+1} \times (3)^{1-n} \times (2^4)^{1-n}} \\ &= \frac{(3)^{8n+2} \times (2)^{4-4n}}{(3)^{9n+1} \times (3)^{1-n} \times (2)^{4-4n}} = 3^{8n+2-9n-1-1+n} = 3^0 = 1 = \text{R.H.S.} \end{aligned}$$

$$\textcircled{2} \frac{125 \times (15)^{n-2} \times (25)^{m+n}}{(75)^n \times (5)^{n+2m}} = \frac{5}{9}$$

Prrof

$$\begin{aligned} \text{L.H.S.} &= \frac{125 \times (15)^{n-2} \times (25)^{m+n}}{(75)^n \times (5)^{n+2m}} = \frac{5^3 \times (3 \times 5)^{n-2} \times (5^2)^{m+n}}{(3 \times 25)^n \times (5)^{n+2m}} \\ &= \frac{5^3 \times 3^{n-2} \times 5^{n-2} \times 5^{2m+2n}}{3^n \times 5^{2n} \times 5^{n+2m}} \\ &= 5^{3+n-2+2m+2n-2n-n-2m} \times 3^{n-2-n} = 5^1 \times 3^{-2} = \frac{5}{9} = \text{R.H.S.} \end{aligned}$$

Example (3)

Find in $\mathbb{R}$ the S.S.:

- | | | |
|------------------------------|--|--------------------------------------|
| $\textcircled{1} x^4 = 16$ | solution: $\because x = \pm \sqrt[4]{16} = \pm 2$ | $\therefore \text{S.S.} = \{2, -2\}$ |
| $\textcircled{2} x^5 = 243$ | solution: $\because x = \sqrt[5]{243} = 3$ | $\therefore \text{S.S.} = \{3\}$ |
| $\textcircled{3} x^4 = -16$ | solution: $\because x = \pm \sqrt[4]{-16}$ it can't | $\therefore \text{S.S.} = \emptyset$ |
| $\textcircled{4} x^5 = -243$ | solution: $\because x = \sqrt[5]{-243} = -3$ | $\therefore \text{S.S.} = \{-3\}$ |



Example (4)

Find in $\mathbb{R}$ the solution set of each of the following equations:

(1) $2^{x+5} = 8$

Solution: $\because 2^{x+5} = 2^3$
 $\therefore x + 5 = 3$
 $\therefore x = 3 - 5 = -2$
 $\therefore$ The S.S. = $\{-2\}$

(2) $3^{x^2-4} = 1$

Solution: $\because 3^{x^2-4} = 1$
 $\therefore x^2 - 4 = 0 \quad \therefore x^2 = 4$
 $\therefore x = \pm\sqrt{4} \quad \therefore x = \pm 2$
 $\therefore$ The S.S. = $\{2, -2\}$

(3) $8x^3 + 125 = 0$

Solution: $\because 8x^3 + 125 = 0$
 $\therefore 8x^3 = -125$
 $\therefore x^3 = \frac{-125}{8}$
 $\therefore x = \sqrt[3]{\frac{-125}{8}} = -\frac{5}{2}$
 $\therefore$ The S.S. = $\{-\frac{5}{2}\}$

(4) $2^{x-2} = 5^{x-2}$

Solution: $\because 2^{x-2} = 5^{x-2}$
 $\therefore x - 2 = 0$
 $\therefore x = 2$
 $\therefore$ The S.S. = $\{2\}$

(5) $\left(\frac{2}{3}\right)^{x+5} = \left(3\frac{3}{8}\right)^{-2}$

Solution: $\because \left(\frac{2}{3}\right)^{x+5} = \left(3\frac{3}{8}\right)^{-2}$
 $\therefore \left(\frac{2}{3}\right)^{x+5} = \left(\frac{27}{8}\right)^{-2}$
 $\therefore \left(\frac{2}{3}\right)^{x+5} = \left(\frac{3^3}{2^3}\right)^{-2}$
 $\therefore \left(\frac{2}{3}\right)^{x+5} = \left(\frac{3}{2}\right)^{-6}$
 $\therefore \left(\frac{2}{3}\right)^{x+5} = \left(\frac{2}{3}\right)^6$
 $\therefore x + 5 = 6 \quad \therefore x = 1$
 $\therefore$ The S.S. = $\{1\}$

(6) $\left(\frac{2}{3}\right)^{|x-5|} = \left(3\frac{3}{8}\right)^{-2}$

Solution: $\because \left(\frac{2}{3}\right)^{|x-5|} = \left(3\frac{3}{8}\right)^{-2}$
 $\therefore \left(\frac{2}{3}\right)^{|x-5|} = \left(\frac{27}{8}\right)^{-2}$
 $\therefore \left(\frac{2}{3}\right)^{|x-5|} = \left(\frac{3^3}{2^3}\right)^{-2}$
 $\therefore \left(\frac{2}{3}\right)^{|x-5|} = \left(\frac{3}{2}\right)^{-6}$
 $\therefore \left(\frac{2}{3}\right)^{|x-5|} = \left(\frac{2}{3}\right)^6$
 $\therefore |x - 5| = 6$
 $\therefore x - 5 = 6 \quad \therefore x - 5 = -6$
 $\therefore x = 11 \quad \therefore x = -1$
 $\therefore$ The S.S. = $\{11, -1\}$



(7) $5^{x+2} = x^{x+2}$

Solution: $\therefore 5^{x+2} = x^{x+2}$
 $\therefore x = 5$
 or
 $\therefore x + 2 = 0$
 $\therefore x = -2$
 $\therefore$ The S.S. = $\{5, -2\}$

(8) $x^{x^2-9} = 4^{x^2-9}$

Solution: $\therefore x^{x^2-9} = 4^{x^2-9}$
 $\therefore x = 4$
 or
 $\therefore x^2 - 9 = 0 \quad \therefore x^2 = 9$
 $\therefore x = \pm\sqrt{9} = \pm 3$
 $\therefore$ The S.S. = $\{4, 3, -3\}$

(9) $2^{x-1} \times 5^{1-x} = \frac{4}{25}$

Solution: $\therefore 2^{x-1} \times 5^{1-x} = \frac{4}{25}$
 $\therefore \frac{2^{x-1}}{5^{x-1}} = \frac{2^2}{5^2}$
 $\therefore \left(\frac{2}{5}\right)^{x-1} = \left(\frac{2}{5}\right)^2$
 $\therefore x - 1 = 2$
 $\therefore x = 3$
 $\therefore$ The S.S. = $\{3\}$

(10) $2^{x+1} + 2^{x-1} = 5$

Solution: $\therefore 2^{x+1} + 2^{x-1} = 5$
 $\therefore (2^x \times 2^1) + (2^x \times 2^{-1}) = 5$
 $\therefore 2^x(2^1 + 2^{-1}) = 5$
 $\therefore 2^x = \frac{5}{(2^1 + 2^{-1})} = 2$
 $\therefore x = 1$
 $\therefore$ The S.S. = $\{1\}$

(11) $2^x = 7 \times 2^{x-1} - 40$

Solution: $\therefore 2^x = 7 \times 2^{x-1} - 40$
 $\therefore 2^x = 7 \times 2^x \times 2^{-1} - 40$
 $\therefore 40 = 7 \times 2^x \times 2^{-1} - 2^x$
 $\therefore 40 = 2^x(7 \times 2^{-1} - 1)$
 $\therefore 2^x = \frac{40}{(7 \times 2^{-1} - 1)}$
 $\therefore 2^x = 16$
 $\therefore 2^x = 2^4$
 $\therefore x = 4$
 $\therefore$ The S.S. = $\{4\}$

(12) $5^x + \frac{125}{5^x} = 30$

Solution: $\therefore 5^x + \frac{125}{5^x} = 30 \quad \times 5^x$
 $\therefore 5^{2x} + 125 = 30 \times 5^x$
 Let $5^x = y$
 $\therefore y^2 + 125 = 30y$
 $\therefore y^2 - 30y + 125 = 0$
 $\therefore y = 25 \quad \text{or} \quad \therefore y = 5$
 $\therefore 5^x = 25 \quad \text{or} \quad \therefore 5^x = 5$
 $\therefore 5^x = 5^2 \quad \text{or} \quad \therefore 5^x = 5^1$
 $\therefore x = 2 \quad \text{or} \quad \therefore x = 1$
 $\therefore$ The S.S. = $\{1, 2\}$

**Example (5)****Prove that:**

$$\frac{5 \times 3^{2n} - 4 \times 3^{2n-1}}{2 \times 3^{2n+1} - 3^{2n}} = \frac{11}{15}$$

Prrof

$$\text{L.H.S.} = \frac{5 \times 3^{2n} - 4 \times 3^{2n-1}}{2 \times 3^{2n+1} - 3^{2n}} = \frac{3^{2n} (5 - 4 \times 3^{-1})}{3^{2n} (2 \times 3 - 1)} = \frac{5 - 4 \times 3^{-1}}{2 \times 3 - 1} = \frac{11}{15} = \text{R.H.S.}$$

Example (6)**① Find the S.S. of the equation: $3^{2x} - 12 \times 3^x + 27 = 0$** *Solution*

$$\therefore 3^{2x} - 12 \times 3^x + 27 = 0$$

$$\therefore y^2 - 12y + 27 = 0$$

$$\therefore y = 9$$

$$\therefore 3^x = 3^2$$

$$\therefore x = 2$$

$$\therefore \text{The S.S.} = \{2, 1\}$$

$$\text{or } \therefore y = 3$$

$$\text{or } \therefore 3^x = 3^1$$

$$\text{or } \therefore x = 1$$

$$\text{Let } y = 3^x$$

② Find the S.S. of the equation: $2 \times 4^x - 9 \times 2^x + 4 = 0$ *Solution*

$$\therefore 2 \times 4^x - 9 \times 2^x + 4 = 0$$

$$\therefore 2y^2 - 9y + 4 = 0$$

$$\therefore y = 4$$

$$\therefore 2^x = 2^2$$

$$\therefore x = 2$$

$$\therefore \text{The S.S.} = \{2, -1\}$$

$$\text{or } \therefore y = \frac{1}{2}$$

$$\text{or } \therefore 2^x = 2^{-1}$$

$$\text{or } \therefore x = -1$$

$$\text{Let } y = 2^x$$

**PRACTICE**

Q1: Evaluate 0.2^0 .

Q2: Simplify $\left(\frac{m}{n^{-1}}\right)^{-3} \left(\frac{2m^{-2}}{n^{-2}}\right)^{-3}$.

☐ A $\frac{1}{8m^9n^9}$

☐ B $\frac{m^3}{8n^9}$

☐ C $\frac{8m^3}{n^9}$

☐ D $\frac{m^3n^3}{8}$

☐ E $8m^9n^3$

Q3: Which of the following is equal to $-\frac{10}{9}x^{-2}y^{-7}$?

☐ A $-\frac{10}{9x^7y^2}$

☐ B $-\frac{10x^2y^7}{9}$

☐ C $-\frac{10}{9x^2y^7}$

☐ D $-\frac{9}{10x^2y^7}$

Q4: True or False: The simplified form of $\frac{2x^{-2}}{y^{-3}}$ is $\frac{2y^3}{x^2}$.

☐ A True

☐ B False

Q5: True or False: The simplified form of $\frac{x^{-4}}{x^{-2}}$ is $\frac{1}{x^2}$.

☐ A True

☐ B False



Q6: Simplify the expression $\left(2x^{\frac{1}{5}}y^{\frac{1}{7}}\right)^2$.

- ☐ A $4x^{\frac{2}{5}}y^{\frac{2}{7}}$
- ☐ B $3x^{\frac{1}{5}}y^{\frac{2}{7}}$
- ☐ C $2x^{\frac{2}{5}}y^{\frac{2}{7}}$
- ☐ D $2x^{\frac{2}{5}}y^2$
- ☐ E $2x^{\frac{1}{5}}y^{\frac{2}{7}}$

Q7: Simplify the expression $\left(\frac{x^8}{y^{-4}}\right)^{\frac{1}{2}}$.

- ☐ A x^4y^{-2}
- ☐ B $x^{-4}y^2$
- ☐ C x^8y^4
- ☐ D $x^{-4}y^{-2}$
- ☐ E x^4y^2

Q8: Simplify the expression $\left(\frac{t^{\frac{3}{8}}v^{\frac{-5}{4}}}{t^{\frac{2}{3}}v^{\frac{1}{2}}}\right)^{\frac{-2}{3}}$.

- ☐ A $t^{\frac{7}{6}}v^{\frac{7}{36}}$
- ☐ B $t^{\frac{-7}{36}}v^{\frac{-7}{6}}$
- ☐ C $t^{\frac{7}{3}}v^{\frac{7}{3}}$
- ☐ D $t^{\frac{7}{36}}v^{\frac{7}{6}}$
- ☐ E $t^{\frac{3}{2}}v^{\frac{1}{6}}$



Exercises 2 - 1



- ① Simplify $\frac{\sqrt{8} \times 4^{-1} \times 2^{-\frac{3}{2}}}{6^{-2} \times 3^2}$
- ② Show when the relation $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$ is true for all real values of a, b?
- ③ Complete each if the following:
 - a) $(8)^{\frac{2}{3}}$ in the simplest form
 - b) $(6\frac{1}{4})^{\frac{3}{2}}$ in the simplest form
 - c) $(\frac{16}{625})^{-\frac{3}{4}}$ in the simplest form
 - d) $\sqrt[3]{(3\frac{3}{8})^{-1}}$ in the simplest form
 - e) $(5^2 - 3^2)^{\frac{1}{2}}$ in the simplest form
- ④ Choose the correct answer from those given:
 - a) If $5^x = 2$ then $25^x =$ (10 , 625 , 4 , 2)
 - b) $(2^7 \div 2^5)^{-\frac{1}{2}} =$ (2 , -2 , $\frac{1}{2}$, $-\frac{1}{2}$)
 - c) If $x^{\frac{3}{2}} = 64$ then $x =$ (512 , 16 , 4 , 2)
 - d) Which of the given is not equal to $(\sqrt[3]{x^4})$ $((\sqrt[5]{x})^4 , \sqrt[4]{x^5} , x^{\frac{4}{5}} , (x^{\frac{1}{5}})^4)$
 - e) If $4x^5 = 128$ then $x =$ (4 , ± 2 , 2 , -2)
 - f) The real roots of the equation $(x - 2)^4 = 16$ are ($\{0\}$, $\{4\}$, $\{8\}$, $\{0, 4\}$)
 - g) If $3^a = 4^b$ then $9^{\frac{a}{b}} + 16^{\frac{b}{a}} =$ (7 , 12 , 20 , 25)
- ⑤ **Find the error:**
 - a) $-9 = (-9)^{\frac{2}{2}} = \sqrt{(-9)^2} = \sqrt{81} = 9$
 - b) If $x^4 = 81$ then $x = \sqrt[4]{81}$ $\therefore x = 3$



- ⑥ **Geometry:** If the length of the radius (r) of the sphere is given by the relation $r = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$.

where (V) is the volume of the sphere find the increase in the radius length when the volume changes from $\frac{32}{3}\pi$ to 36π cubic units.

- ⑦ Find the solution set of each of following equations:

a $x^{\frac{5}{2}} = \frac{1}{32}$

b $x^4 = 81$

c $\sqrt[3]{(x-1)^5} = 32$

d $(x^2 - 5x + 9)^{\frac{5}{2}} = 243$

e $x^{\frac{4}{5}} - 5x^{\frac{2}{5}} + 4 = 0$

f $x + 15 = 8\sqrt{x}$

g $\sqrt[3]{x^2} - 25\sqrt[3]{x} - 54 = 0$

h $(2x - 1)^4 = (x + 3)^4$

- ⑧ If $x^{\frac{3}{2}} = 3y^{\frac{2}{3}} = 27$ find the value of $x + y$

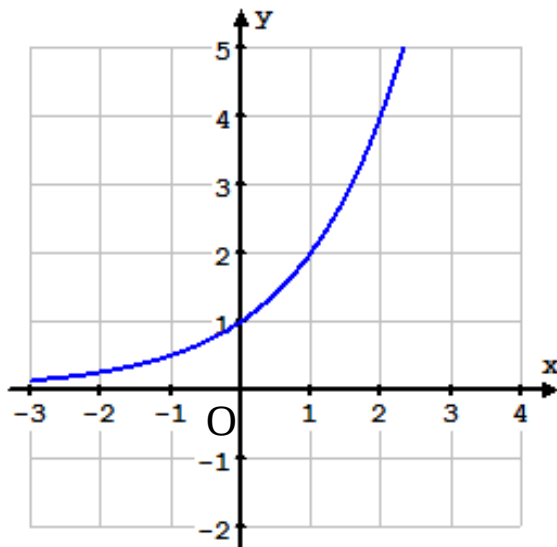
**Lesson (2)****Exponential Function & its Applications****Lesson objectives****Related Links**

- ✚ Determine the Exponential function.
- ✚ Recognize the graphical representation of the exponential function.
- ✚ Identify Properties of the exponential function.

Important Notes:

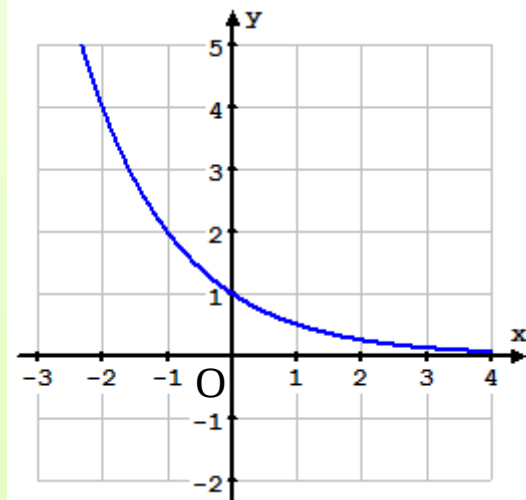
■ If a is a positive real number $\neq 1$, then the function $f : \mathbb{R} \rightarrow \mathbb{R}^+$ where $f(x) = a^x$ is called " **an exponential function** " whose base is " a "

$$f : \mathbb{R} \rightarrow \mathbb{R}^+, f(x) = a^x : a > 1$$



- The function is increasing
- The range is $\mathbb{R}^+$

$$f : \mathbb{R} \rightarrow \mathbb{R}^+, f(x) = a^x : 0 < a < 1$$



- The function is decreasing
- The range is $\mathbb{R}^+$



Example (1)

Determine which of the following functions represent exponential function:

① $f(x) = x^2$	Exponential	or	Not-Exponential
② $f(x) = 2^x$	Exponential	or	Not-Exponential
③ $f(x) = \frac{3}{x+1}$	Exponential	or	Not-Exponential
④ $f(x) = x^3 - 1$	Exponential	or	Not-Exponential
⑤ $f(x) = \left(\frac{3}{4}\right)^{x-1}$	Exponential	or	Not-Exponential
⑥ $f(x) = 2^{3x}$	Exponential	or	Not-Exponential

Example (2)

If: $f(x) = 3^x$, then: Complete each of the following:

- ① $f(2) = 3^2 = 9$ ② $f(x+2) = 3^{x+2} = 3^x \times 3^2 = 9 \times f(x)$
 ③ $f(x) \times f(-x) = 3^x \times 3^{-x} = 3^{x+(-x)} = 3^0 = 1$

Properties of the exponential function $f(x) = a^x$, $a > 0$, $a \neq 1$

- 1) The domain of $f(x) = a^x$ is $\mathbb{R}$ and its range is $]0, \infty[$
- 2) If $a > 1$ then the function is increasing on its domain and named by exponential growth
if $0 < a < 1$ then the function is decreasing on its, domain and it is named by exponential decay.
- 3) The curve of $f(x) = a^x$ passes through the point $(0, 1)$ for all $a > 0$, $a \neq 1$
- 4) $f(x) = a^x$ is One - to - One function
- 5) The curve of the function $f(x) = a^x$ is image of the curve $f(x) = \left(\frac{1}{a}\right)^x$ by reflection in y-axis
- 6) $a^x \longrightarrow \infty$ when $x \longrightarrow \infty$ if $a > 1$
 $a^x \longrightarrow 0$ when $x \longrightarrow \infty$ if $0 < a < 1$

**Example (3)**

If $f : \mathbb{R} \rightarrow \mathbb{R}^+, f(x) = 7^x$

First: Find the value of each of: $f(-3), f(-2), f\left(\frac{1}{2}\right)$ and $f(0)$

Second: prove that: $f(a) \times f(b) = f(a+b)$

Third: If $f(x) + f(x-1) = 56$ find the value of x

Solution

First: $f(-3) = 7^{-3} = \frac{1}{7^3} = \frac{1}{343}$ $f(-2) = 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$

$$f\left(\frac{1}{2}\right) = 7^{\frac{1}{2}} = \sqrt{7} \qquad f(0) = 7^0 = 1$$

Second: **L.H.S. = $f(a) \times f(b) = 7^a \times 7^b = 7^{(a+b)} = f(a+b) = \text{R.H.S.}$**

Third: $\therefore f(x) + f(x-1) = 56$ $\therefore 7^x + 7^{x-1} = 56$
 $\therefore 7^x(1 + 7^{-1}) = 56$ $\therefore 7^x = \frac{56}{(1 + 7^{-1})} = 49$
 $\therefore 7^x = 7^2$ $\therefore x = 2$

Example (4)

If $f : \mathbb{R} \rightarrow \mathbb{R}^+, f(x) = 3^x$ prove that: $\frac{f(x+5) + f(x+3)}{f(x+3) + f(x+1)} = f(2)$

Solution

L.H.S. $= \frac{f(x+5) + f(x+3)}{f(x+3) + f(x+1)} = \frac{3^{x+5} + 3^{x+3}}{3^{x+3} + 3^{x+1}} = \frac{\cancel{3^x}(3^5 + 3^3)}{\cancel{3^x}(3^3 + 3^1)}$
 $= 9 = 3^2 = f(2) = \text{R.H.S.}$

**Example (5)**

If: $f(x) = 7^x$ Find the value of x if: $f(2x - 1) + f(2x + 1) = \frac{50}{49}$

Solution

$$\therefore f(2x - 1) + f(2x + 1) = \frac{50}{49}$$

$$\therefore 7^{2x-1} + 7^{2x+1} = \frac{50}{49}$$

$$\therefore 7^{2x} = \frac{50}{49} \div (7^{-1} + 7^1)$$

$$\therefore 2x = -1$$

$$\therefore 7^{2x}(7^{-1} + 7^1) = \frac{50}{49}$$

$$\therefore 7^{2x} = \frac{1}{7} = 7^{-1}$$

$$\therefore x = \frac{-1}{2}$$

Example (6)

If: $f(x) = 5^x$, then prove that: $\frac{4f(x+2) - 19f(x)}{2f(x+1) + 8f(x)} = \frac{9}{2}$

Solution

$$\text{L.H.S.} = \frac{4f(x+2) - 19f(x)}{2f(x+1) + 8f(x)} = \frac{4 \times 5^{x+2} - 19 \times 5^x}{2 \times 5^{x+1} + 8 \times 5^x}$$

$$= \frac{\cancel{5^x}(4 \times 5^2 - 19)}{\cancel{5^x}(2 \times 5^1 + 8)} = \frac{9}{2} = \text{R.H.S.}$$

Example (7)

If: $f(x) = 7^x$, $f(2x + 2) + f(2x + 1) = 392$, then find x

Solution

$$\therefore f(2x + 2) + f(2x + 1) = 392$$

$$\therefore 7^{2x+2} + 7^{2x+1} = 392$$

$$\therefore 7^{2x} = 392 \div (7^2 + 7^1)$$

$$\therefore 2x = 1$$

$$\therefore 7^{2x}(7^2 + 7^1) = 392$$

$$\therefore 7^{2x} = 7$$

$$\therefore x = \frac{1}{2}$$



PRACTICE (1)

Q1: Write an exponential equation in the form $y = b^x$ for the numbers in the table.

x	2	4	5
y	$\frac{9}{16}$	$\frac{81}{256}$	$\frac{243}{1,024}$

☐ A $y = \left(\frac{3}{4}\right)^x$

☐ C $y = \left(\frac{9}{32}\right)^x$

☐ E $y = (x)^{\frac{3}{4}}$

☐ B $y = \left(\frac{9}{16}\right)^x$

☐ D $y = (x)^{\frac{9}{16}}$

Q2: Write an exponential equation in the form $y = a(b^x)$ for the numbers in the table.

x	0	1	2	3
y	18	6	2	$\frac{2}{3}$

☐ A $y = 2(3^x)$

☐ C $y = 3(2^x)$

☐ E $y = 2x^3$

☐ B $y = 18\left(\frac{1}{3}\right)^x$

☐ D $y = \frac{1}{3}(18^x)$

Q3: A start-up company noticed that the number of those who use its product doubles every month. This month, they had 4 000 users. Assuming this trend continues, write an equation that can be used to calculate U , the number of users in m months' time.

☐ A $U = 4\,000(3)^m$

☐ B $U = 4\,000(m)^3$

☐ C $U = 4\,000(m)^2$

☐ D $U = 4\,000(2)^m$

☐ E $U = 4\,000(2m)$

Q4: The number of marine organisms in a pool, y , after n weeks is given by the formula $y = 4\,344\left(\frac{1}{2}\right)^{n-1}$. After how many weeks are there 1 086 marine organisms in the pool?



Q5: The population of a town doubles every 50 years. How long does it take for the population to triple? Round your answer to two decimal places.

Q6: A microorganism reproduces by binary fission, where every hour each cell divides into two cells. Given that there were 15 141 cells to begin with, determine how many cells there were after 5 hours.

Example (8)

If: $f(x) = 2^x$, then find the S.S. of: $f(2x) - 6f(x) + f(3) = 0$

Solution

$$\therefore f(2x) - 6f(x) + f(3) = 0$$

$$\therefore 2^{2x} - 6 \times 2^x + 8 = 0$$

$$\therefore y^2 - 6y + 8 = 0$$

$$\therefore y = 4$$

$$\therefore 2^x = 4$$

$$\therefore 2^x = 2^2$$

$$\therefore x = 2$$

&

&

&

$$x = 1$$

$$\therefore 2^{2x} - 6 \times 2^x + 2^3 = 0$$

$$\text{Let } y = 2^x$$

$$y = 2$$

$$\therefore 2^x = 2$$

$$\therefore 2^x = 2$$

$$\therefore \text{The S.S.} = \{1, 2\}$$

Example (9)

Graph the function $f: \mathbb{R} \rightarrow \mathbb{R}^+, f(x) = 2^x$ taking $x \in [-3, 4]$ and from the graph find:

① $f(1.5), f\left(\frac{1}{2}\right)$

② The value of x when $f(x) = 10$

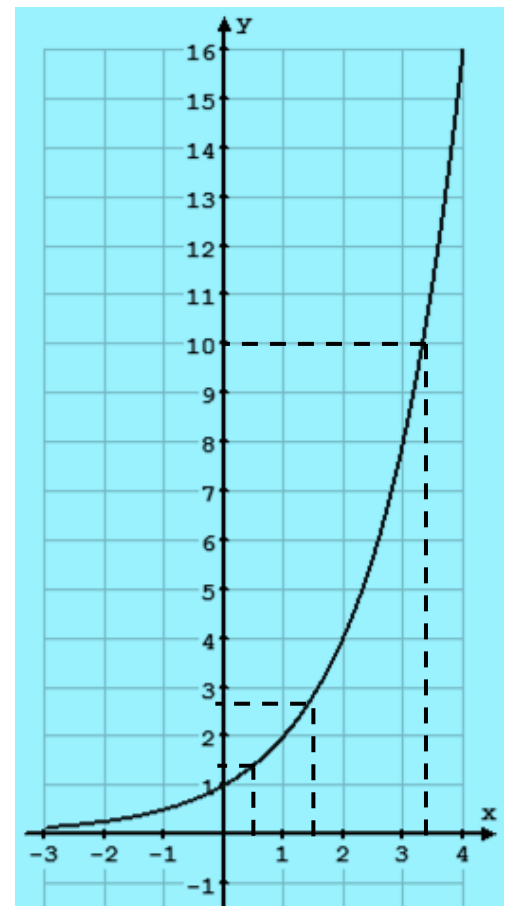
Solution

x	-3	-2	-1	0	1	2	3	4
$y = 2^x$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16

From the graph:

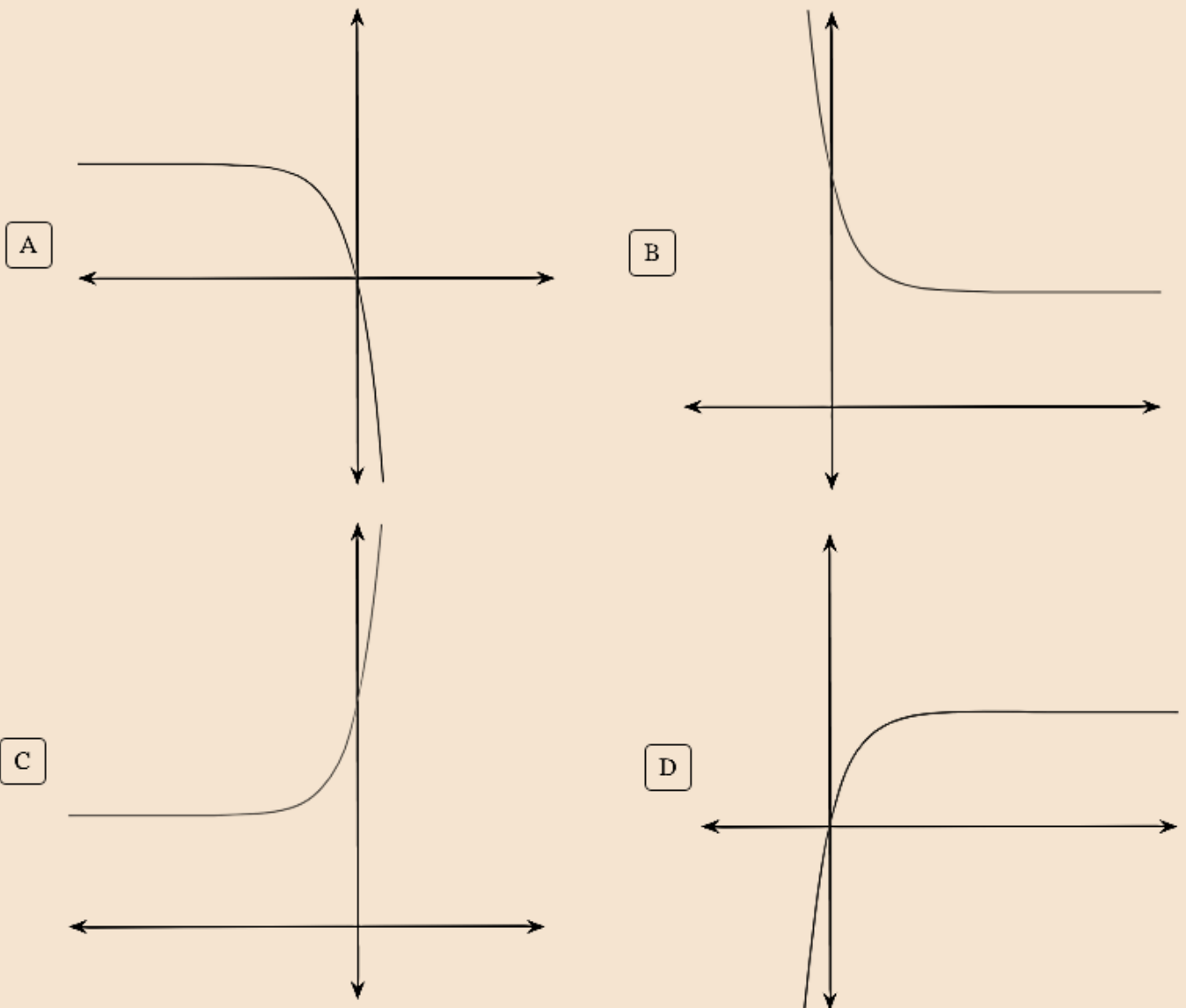
① $f(1.5) \approx 2.8$ $f\left(\frac{1}{2}\right) \approx 1.4$

② If $f(x) = 10$ $\therefore x \approx 3.3$



**PRACTICE (2)**

Q1: Which graph demonstrates exponential growth?



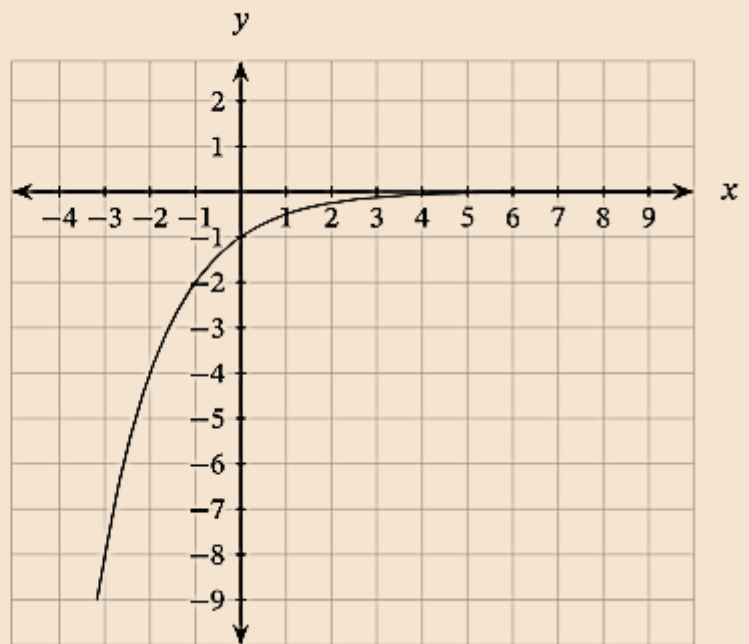
Q2: Determine the point at which the graph of the function $f(x) = 6^x$ intersects the y -axis.

- ☐ A (6, 0)
- ☐ B (1, 0)
- ☐ C (0, 1)
- ☐ D (0, 6)



Q3: Determine the function represented by the graph shown.

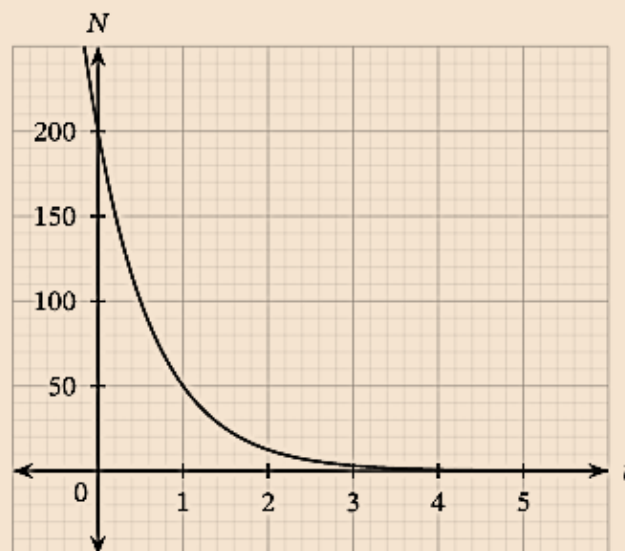
- ☐ A $y = -2^x$
- ☐ B $y = 2^x$
- ☐ C $y = 2^{-x}$
- ☐ D $y = -2^{-x}$



Q4: The graph of the function $f(x) = a^x$ passes through the point $(3, 1)$. What is the value of a ?

Q5: Which of the following expressions does NOT describe the shown graph?

- ☐ A $N = 200 \cdot e^{-1.39t}$
- ☐ B $N = 200 \cdot (4)^{-t}$
- ☐ C $N = 200 \cdot \left(\frac{1}{2}\right)^{2t}$
- ☐ D $N = 200 \cdot \left(\frac{1}{4}\right)^t$
- ☐ E $N = 200 \cdot \left(\frac{1}{2}\right)^{\frac{t}{2}}$





Applications tends to equations in the form $a^x = b$

Growth and Decay

In our daily life there are a lot of phenomena expressing growth and decay by time such as the study of population, bacteria, viruses, radiation substances, electricity and temperature.

In algebra, there are two functions, representing the growth and decay which are exponential growth function and exponential decay function.

First: Exponential growth

We can use the function f , such that:

$f(t) = a(1 + r)^t$ to represent the exponential growth with a constant percentage during constant intervals of time.

Where: **(t)** is the time.

(a) is the initial value.

(r) is the growth percentage per interval of time.

If principal P is deposited in one of the banks at interest rate r (percentage) and compounded n times per year for a period of t years, then the accumulated value

A is given by: $A = P \left(1 + \frac{r}{n}\right)^{nt}$

Example (10)

A man deposited a capital of 5000 L.E in one of the banks with annual compound interest 8%.

Find the sum of the capital after 10 years in each of the following:

- (a) The interest compounded annually.
- (b) The interest compounded quarter annually.
- (c) The interest compounded monthly.

Solution

Use the relation: $A = P \left(1 + \frac{r}{n}\right)^{nt}$

(a) The interest compounded annually: $\therefore n = 1$

$$\therefore A = 5000 \left(1 + \frac{0.08}{1}\right)^{10 \times 1} = \text{L.E. } 10794.62$$

(b) The interest compounded quarter annually: $\therefore n = 4$

$$\therefore A = 5000 \left(1 + \frac{0.08}{4}\right)^{10 \times 4} = \text{L.E. } 11040.2$$

(c) The interest compounded monthly: $\therefore n = 12$

$$\therefore A = 5000 \left(1 + \frac{0.08}{12}\right)^{10 \times 12} = \text{L.E. } 11098.2$$



Second: Exponential decay

We can use the function f , such that:

$f(t) = a(1 - r)^t$ to represent the exponential decay with a constant percentage during constant intervals of time.

Where: **(t)** is the time.

(a) is the initial value.

(r) is the growth percentage per interval of time.

Example (11)

Kareem bought a car at 120000 L.E, and its price decreases at the rate of 12% per a year.

(a) Write the exponential function which represents the price of the car after t years.

(b) Estimate to the nearest pound the price of the car after 6 years.

Solution

Use the relation: $C = a(1 - r)^t$

(a) The exponential function which represents the price of the car after t years:

$$\therefore C = a(1 - r)^t$$

$$\therefore f(t) = 120000(1 - 0.12)^t$$

(b) The price of the car after 6 years:

$$\therefore f(t) = 120000(1 - 0.12)^t$$

$$\therefore f(6) = 120000(1 - 0.12)^6 = \text{L.E. } 55728.49$$

PRACTICE (3)

Q1: Is the exponential function $y = 4(1.21)^x$ growing or decaying?

☐ A decaying

☐ B growing

Q2: The population, N , of a city in year t is given by the formula $N = 10^4(2.6)^{t-2001}$. Determine the year in which the population of the city was 8 million.



Q3: Karim invests in a savings account. After ten years, the value of his investment had doubled. What was the annual rate of interest? Give your answer to one decimal place.

Q4: A population of bacteria in a petri dish t hours after the culture has started is given by $P = 2400 \cdot e^{0.084t}$. Amira says this means that the growth rate is 8.4% per hour. Her friend Engy, however, says that the hourly growth rate is 8.76%. Who is right?

- ☐ A Engy
- ☐ B Amira

Q5: The value of a car falls by 36% over 2 years. By considering a suitable exponential function, find the equivalent annual rate of depreciation that would produce the same fall in value over two years.

- ☐ A 25%
- ☐ B 20%
- ☐ C 60%
- ☐ D 40%
- ☐ E 12%

Q6: The price of an article increases by 9% every year. Given that its original price was 3,652 LE, determine its price after 7 years. Give your answer to the nearest pound.

Q7: A man deposited 1 078 LE in a bank account with an interest rate of 7% per year. Determine how much money was in the account 5 years later, given that the interest was compounded monthly. Give your answer to the nearest pound.

Q8: A man deposited 5,094 LE in a bank account with an interest rate of 7% per year. Determine how much money was in the account 19 years later, given that the interest was compounded annually. Give your answer correct to two decimal places.



Exercises 2 - 2



- 1 Complete each of the following:
 - a The function f such that $f(x) = a^x$ is an exponential function if a, x
 - b The exponential function g where $g(x) = 3^{x-1}$ its base is
 - c The function K where $K(x) = \left(\frac{1}{2}\right)^{x+1}$ is not exponential because
 - d The coordinates of the point of intersection of the curve of the function $f(x) = a^x$ with the straight line $x = 0$ is the point (.....,
 - e The equation of the line of symmetry of the graph of the two functions f and g where, $g(x) = 3^x$, $g(x) = \left(\frac{1}{3}\right)^x$ is
- 2 Choose the correct answer from those given :
 - a The exponential function of base a is increasing if
 - (a) $a > 0$
 - (b) $a > 1$
 - (c) $0 < a < 1$
 - (d) $a = 1$
 - b The exponential function of base a is decreasing if :
 - (a) $a > 0$
 - (b) $a < 0$
 - (c) $0 < a < 1$
 - (d) $-1 < a < 0$
 - c The exponential function $f(x) = a^x$, $a > 1$ its curve approaches:
 - (a) the x -axis (positive direction)
 - (b) the x -axis (negative direction)
 - (c) the y -axis (positive direction)
 - (d) the y -axis (negative direction)
 - e In the exponential function $g(x) = a^x$, $(0 < a < 1)$ then $0 < a^x < 1$ when $x \in$
 - (a) $]0, \infty[$
 - (b) $]-\infty, 0]$
 - (c) $]1, \infty[$
 - (d) $]-\infty, 1]$
- 3 Show which of the following is an exponential function then determine the base and the power of each:-
 - a $f(x) = 2x^3$
 - b $f(x) = \frac{2}{3}(5)^x$
 - c $f(x) = \frac{1}{x-1}$
 - d $f(x) = 3x^2 - 1$
 - e $f(x) = \left(\frac{2}{3}\right)^{x-1}$
 - f $f(x) = (-7)^x$



- ④ Represent graphically each of the following functions then find the domain and the range of each. Also determine which is increasing and which is decreasing:
- a $f(x) = 3^x$ b $f(x) = \left(\frac{1}{2}\right)^x$ c $f(x) = -3(2)^x$ d $f(x) = 2^{x+1} + 1$
 e $f(x) = \left(\frac{1}{2}\right)^{x+2} - 2$ f $f(x) = 2\left(\frac{2}{3}\right)^{x-1} + 1$ g $f(x) = -\left(\frac{1}{2}\right)^{2x} + \frac{3}{4}$
- ⑤ **Saving:** Ziad deposit 80000 L.E in a bank which gives an annual interest of 10.5%. Find the total amount of money after 10 years given that the total amount is given by $C = a(1+r)^t$ where t is the number of years, a the starting amount, r the annual interest
- ⑥ **Communication:** The number of the land lines telephone decreases in a city as a result of the proliferation of the mobile phones at the rate of 10% yearly. If the number of the land lines in a year was 54000 lines write the exponential function which represents the number of lines after t years then estimate the number of lines after 3 years.
- ⑦ **Investment:** The number of cows in a cattle farm is 80 cows and the reproduction rate of these cows is 18% annually. Find the number of cows after 4 years.
- ⑧ **Population:** The number of population in a city of A.R.E reached 4.6 million people with an average increase 4% annually.
1st: Write the exponential growth function after t years.
2nd: Estimate the number of population after 5 years.
- ⑨ **Sport:** The number of spectators of a football team decreases at the rate of 4% each match as a result of recurrent loss in a championship, and if the number of spectators in the first match was 36400. Write the exponential function which represents the number of spectators (y) in the match (t) then estimate the number of fans in the tenth match.



Lesson (3)

Fractional Exponents

Lesson objectives

Related Links



- Solve Exponential Equation.
- Identify Graphical Solution.

Important Notes:

- For every $n \in \mathbb{Z}^+$, $a, b \in \mathbb{R}^+$ we have: $b = a^{\frac{1}{n}} \Leftrightarrow b^n = a$
- For all $x > 0$, $n \in \mathbb{Z}^+$, $m \in \mathbb{Z}$ we have: $x^{\frac{m}{n}} = \sqrt[n]{x^m}$

Example (1)

Put the following in the simplest form:

$$(1) (81)^{\frac{1}{2}} \times (81)^{\frac{1}{4}} = (81)^{\frac{1}{2} + \frac{1}{4}} = (81)^{\frac{3}{4}} = (3^4)^{\frac{3}{4}} = 3^3 = 27$$

$$(2) \left(2\frac{1}{4}\right)^{1.5} \times \sqrt[5]{\sqrt{1024}} = \left(\frac{9}{4}\right)^{\frac{3}{2}} \times \left[(1024)^{\frac{1}{2}}\right]^{\frac{1}{5}} = \left(\frac{3^2}{2^2}\right)^{\frac{3}{2}} \times \left[(2^{10})^{\frac{1}{2}}\right]^{\frac{1}{5}} \\ = \left[\left(\frac{3}{2}\right)^2\right]^{\frac{3}{2}} \times \left[(2^{10})^{\frac{1}{2}}\right]^{\frac{1}{5}} = \left(\frac{3}{2}\right)^3 \times 2 = \frac{27}{4}$$

$$(3) \frac{(625)^{\frac{3}{4}x} \times (27)^{x-\frac{1}{3}}}{(225)^{1.5x} \times \sqrt{25}} = \frac{(5^4)^{\frac{3}{4}x} \times (3^3)^{x-\frac{1}{3}}}{(15^2)^{\frac{3}{2}x} \times 5} = \frac{5^{3x} \times 3^{3x-1}}{(15)^{3x} \times 5} = \frac{5^{3x} \times 3^{3x-1}}{3^{3x} \times 5^{3x} \times 5} = \frac{3^{-1}}{5} = \frac{1}{15}$$

$$(4) \frac{\sqrt[8]{25} \times (\sqrt[4]{35})^{-1} \times 7^3 \times (225)^{\frac{3}{8}}}{(21)^{\frac{3}{4}} \times 5^{\frac{3}{4}}} = \frac{5^{\frac{2}{8}} \times 5^{\frac{-1}{4}} \times 7^{\frac{-1}{4}} \times 7^3 \times (15^2)^{\frac{3}{8}}}{3^{\frac{3}{4}} \times 7^{\frac{3}{4}} \times 5^{\frac{3}{4}}} \\ = \frac{5^{\frac{1}{4}} \times 5^{\frac{-1}{4}} \times 7^{\frac{-1}{4}} \times 7^3 \times 5^{\frac{3}{4}} \times 3^{\frac{3}{4}}}{3^{\frac{3}{4}} \times 7^{\frac{3}{4}} \times 5^{\frac{3}{4}}} = 7^{\frac{-1}{4} + 3 - \frac{3}{4}} = 7^2 = 49$$



Example (2)

Find in $\mathbb{R}$ the solution set of each of the following equations:

(1) $x^{\frac{3}{4}} = 27$

Solution: $\because x^{\frac{3}{4}} = 27$ $\therefore x = 27^{\frac{4}{3}}$
 $\therefore x = 81$ $\therefore$ The S.S. = $\{81\}$

(2) $2^x \times \sqrt[3]{4} = (\sqrt[3]{16})^{-1}$

Solution: $\because 2^x \times \sqrt[3]{4} = (\sqrt[3]{16})^{-1}$ $\therefore 2^x \times 2^{\frac{2}{3}} = 2^{-\frac{4}{3}}$
 $\therefore 2^x = 2^{-\frac{4}{3}} \div 2^{\frac{2}{3}}$ $\therefore 2^x = 2^{-\frac{4}{3} - \frac{2}{3}} = 2^{-2}$
 $\therefore x = -2$ $\therefore$ The S.S. = $\{-2\}$

(3) $4^{x^2-1} = 8^{-x}$

Solution: $\because 4^{x^2-1} = 8^{-x}$ $\therefore 2^{2x^2-2} = 2^{-3x}$
 $\therefore 2x^2 - 2 = -3x$ $\therefore 2x^2 + 3x - 2 = 0$
 $\therefore x = \frac{1}{2}$ or $\therefore x = -2$ $\therefore$ The S.S. = $\{\frac{1}{2}, -2\}$

(4) $3^{x-2} = (27)^{\frac{1}{x}}$

Solution: $\because 3^{x-2} = (27)^{\frac{1}{x}}$ $\therefore 3^{x-2} = 3^{\frac{3}{x}}$
 $\therefore x - 2 = \frac{3}{x} \times x$
 $\therefore x^2 - 2x = 3$ $\therefore x^2 - 2x - 3 = 0$
 $\therefore x = 3$ or $\therefore x = -1$ $\therefore$ The S.S. = $\{3, -1\}$

(5) $\left(x^{\frac{1}{3}} - 1\right)\left(x^{\frac{2}{3}} + x^{\frac{1}{3}} + 1\right) = 3$

$$x^3 - 1 = (x - 1)(x^2 + x + 1)$$

Solution: $\because \left(x^{\frac{1}{3}} - 1\right)\left(x^{\frac{2}{3}} + x^{\frac{1}{3}} + 1\right) = 3$
 $\therefore (x^{\frac{1}{3}})^3 - 1^3 = 3$
 $\therefore x = 4$

$\therefore x - 1 = 3$
 $\therefore$ The S.S. = $\{4\}$



Example (3)

Find in $\mathbb{R}$ the solution set of each of the following equations:

a $x^{\frac{7}{2}} = 128$

b $(2x + 3)^{\frac{4}{3}} = 81$

c $x^{\frac{4}{3}} - 13x^{\frac{2}{3}} + 36 = 0$

d $\sqrt[3]{x^5} - 31\sqrt[6]{x^5} - 32 = 0$

Solution

a $x^{\frac{7}{2}} = 128$
 $x = (128)^{\frac{2}{7}}$
 $x = 2^2 = 4$
 $x = (2^7)^{\frac{2}{7}}$
 $\therefore$ solution set = $\{4\}$

b $(2x + 3)^{\frac{4}{3}} = 81$
 $((2x + 3)^{\frac{4}{3}})^3 = (3^4)^3$ raise both sides to the power 3
 $(2x + 3)^4 = 3^{12}$
 $2x + 3 = \pm (3^{12})^{\frac{1}{4}} \quad 2x + 3 = \pm 3^3$
Either $2x + 3 = 27 \quad 2x = 24 \quad \therefore x = 12$
Or $2x + 3 = -27 \quad 2x = -30 \quad \therefore x = -15$
 solution set = $\{12, -15\}$

c $x^{\frac{4}{3}} - 13x^{\frac{2}{3}} + 36 = 0$
 $(x^{\frac{2}{3}} - 9)(x^{\frac{2}{3}} - 4) = 0$
either $x^{\frac{2}{3}} - 9 = 0$ **or** $x^{\frac{2}{3}} - 4 = 0$
 $x^{\frac{2}{3}} = 3^2 \quad x^{\frac{2}{3}} = 2^2$
 $x = \pm (3^2)^{\frac{3}{2}} \quad x = \pm (2^2)^{\frac{3}{2}}$
 $x = \pm 27 \quad x = \pm 8$
 solution set = $\{27, -27, 8, -8\}$

d $\sqrt[3]{x^5} - 31\sqrt[6]{x^5} - 32 = 0$
 $x^{\frac{5}{3}} - 31x^{\frac{5}{6}} - 32 = 0$
 $(x^{\frac{5}{6}} - 32)(x^{\frac{5}{6}} + 1) = 0$
either $x^{\frac{5}{6}} - 32 = 0$ **or** $x^{\frac{5}{6}} + 1 = 0$
 $x^{\frac{5}{6}} = 2^5 \quad x^{\frac{5}{6}} = -1$ **refused**
 $x = (2^5)^{\frac{6}{5}}$
 $x = 64$
 solution set = $\{64\}$

**PRACTICE (1)**

Q1: Simplify $\frac{(25)^{\frac{3}{2}x} \times (8)^{x-\frac{5}{3}}}{(100)^{\frac{3}{2}x} \times \sqrt{100}}$.

☐ A $\frac{5}{16}$

☐ B $\frac{1}{320}$

☐ C 320

☐ D $\frac{16}{5}$

Q2: Determine the simplest form of $\frac{(16)^{\frac{3}{2}x} \times 27^{x-\frac{1}{3}}}{(144)^{\frac{3}{2}x} \times \sqrt{81}}$.

☐ A $\frac{1}{27}$

☐ B 3

☐ C $\frac{1}{3}$

☐ D 27

Q3: Simplify $\frac{(36)^{x+\frac{1}{2}} \times 8^{x+1}}{6^{x-1} \times (12)^{x+5} \times (\sqrt{16})^{x-1}}$.

☐ A 864

☐ B 216

☐ C $\frac{1}{216}$

☐ D $\frac{1}{864}$



Q4: Simplify $\frac{4^{3n+3} \times 25^{1-3n}}{2^{9n+3} \times 50^{1-3n}}$.

- ☐ A 4
- ☐ B 256
- ☐ C $\frac{1}{256}$
- ☐ D $\frac{1}{4}$

Q5: Simplify $\frac{45^{18n} \times (63)^{-9n} \times 3^{9n}}{225^{9n} \times (21)^{-9n}}$.

- ☐ A 3^{18n}
- ☐ B 3^n
- ☐ C 3^{9n}
- ☐ D 9^n

Q6: Simplify $\frac{(\sqrt{2})^{8n+10} \times 30^{8n+3}}{(\sqrt{2})^{8n} \times 6^{8n} \times 5^{8n+4}}$.

- ☐ A $\frac{6912}{5}$
- ☐ B $\frac{5}{216}$
- ☐ C $\frac{5}{6912}$
- ☐ D $\frac{216}{5}$



Q7: Simplify $\frac{125^{n-1} \times 25^{-n}}{25 \times 5^n}$.

- ☐ A 150
- ☐ B $\frac{1}{150}$
- ☐ C 3 125
- ☐ D $\frac{1}{3\,125}$

Q8: Simplify $\frac{a^{x+9} + a^{x+8} + a^{x+7}}{a^{x-2} + a^{x-3} + a^{x-4}}$.

- ☐ A a^8
- ☐ B a^3
- ☐ C 1
- ☐ D a^{11}
- ☐ E ax

Q9: Calculate $\frac{7 \times 5^{5n} - 9 \times 5^{5n+2}}{3 \times 5^{5n+1} - 5^{5n}}$, giving your answer in its simplest form.

- ☐ A $-\frac{109}{7}$
- ☐ B $\frac{5}{3}$
- ☐ C $\frac{5}{14}$
- ☐ D 5
- ☐ E $-\frac{218}{3}$

**PRACTICE (2)**

Q1: Simplify $\sqrt[3]{-0.064a^{30}}$.

- ☐ A $-0.064a^{10}$
- ☐ B $0.4a^{10}$
- ☐ C $-0.4a^{30}$
- ☐ D $-0.4a^{10}$
- ☐ E $0.04a^{10}$

Q2: Which of the following is equal to $\sqrt{(x+y)^{58}}$?

- ☐ A $-(x+y)$
- ☐ B $|(x+y)^{29}|$
- ☐ C $|(x+y)|$
- ☐ D $(x+y)^{56}$
- ☐ E $-(x+y)^{29}$

Q3: Given that $x < 0$, simplify $\sqrt{x^2} - \sqrt[3]{x^3} - \sqrt{x^2 - 2x + 1} + 1 = \underline{\hspace{2cm}}$.

- ☐ A $-x$
- ☐ B 0
- ☐ C x
- ☐ D -1



Exercises 2 - 3



- 1 Choose the correct answer:
 - a If $2^{x+1} = 8$, then $x =$
 - (a) 1
 - (b) 2
 - (c) 4
 - (d) 3
 - b If $5^{x-1} = 4^{x-1}$, then $x =$
 - (a) 5
 - (b) 1
 - (c) -1
 - (d) 0
 - c $(\frac{1}{2})^{a^2-a-2} = 1$ where $a > 0$, then $a =$
 - (a) 1
 - (b) -3
 - (c) 2
 - (d) 3
- 2 Find the solution set of each of the following equations:
 - a $2^{x+1} = 4$
 - b $3^{x-1} = \frac{1}{9}$
 - c $7^{x-2} = 1$
 - d $5^{x+3} = 4^{x+3}$
 - e $(3\sqrt{3})^{|x|} = 27$
 - f $3^{x+3} - 3^{x+2} = 162$
 - g $5^{2x} + 25 = 26 \times 5^x$
 - h $2^x + 2^{5-x} = 12$
 - i $(\frac{1}{2})^{x+1} + (\frac{1}{2})^{x+3} + (\frac{1}{2})^{x+5} = 84$
- 3 Find the S.S of the two equations:
 $3^x \times 5^y = 75$, $3^y \times 5^x = 45$
- 4 If $f_1(x) = 3^x, f_2(x) = 9^x$ find the value of x which satisfy $f_1(2x-1) + f_2(x+1) = 756$
- 5 If $f(x) = 7^{x+1}$ find x which satisfy $f(2x-1) + f(x-2) = 50$
- 6 find graphically the solution set of the equation:
 - a $3^{x-2} = 3 - x$
 - b $2^x = 2x$
- 7 **Creative thinking:** If $x^3 = y^2$ and $x^{n+1} = y^{n-1}$ find the value of n ?
- 8 **Numbers:** If the sum of $2 + 4 + 8 + 16 + \dots + 2^n$ is given by the relation $s_n = 2(2^n - 1)$
 - a Find the sum of the first ten numbers in this pattern
 - b Find the number of terms of this pattern starting from the first term to give the sum 131070
- 9 Solve each of the following equations
 - a $3^{x^2-42} = (\frac{1}{3})^x$
 - b $7^{2-x} + 7^{-x} = 50$

**Lesson (2)****Finding the limit of a function at a point****Lesson objectives****Related Links**

- ✚ Identify the limit of the polynomial function.
- ✚ Solve some of limits theorems.
- ✚ Find the limit of function using the long division.

Important Notes:

$\lim_{x \rightarrow a} f(x)$ can be equal to:

- ① a real number $\Rightarrow$ « the limit of the function is this real number »
- ② $\frac{\text{a real number} \neq 0}{\text{zero}}$ $\Rightarrow$ « the function has no limit »
- ③ $\frac{\text{zero}}{\text{zero}}$ $\Rightarrow$ « then $(x - a)$ is a factor of both terms the numerator and the denominator »
(Unspecified quantity)

Example (1)**By Substitution**

Find the limit of the following functions:

(1) $\lim_{x \rightarrow 3} (2x - 7)$

Solution: $\lim_{x \rightarrow 3} (2x - 7) = 2(3) - 7 = -1$

(2) $\lim_{x \rightarrow 5} (x^2 - 25)$

Solution: $\lim_{x \rightarrow 5} (x^2 - 25) = (5)^2 - 25 = 0$

(3) $\lim_{x \rightarrow -1} (x^2 - x - 5)$

Solution: $\lim_{x \rightarrow -1} (x^2 - x - 5) = (-1)^2 - (-1) - 5 = -3$

(4) $\lim_{x \rightarrow 1} \frac{x^2 + 3x + 2}{2x - 2}$

Solution: $\lim_{x \rightarrow 1} \frac{x^2 + 3x + 2}{2x - 2} = \frac{(1)^2 + 3(1) + 2}{2(1) - 2} = \frac{6}{0} = \infty$ or has no limit

(5) $\lim_{x \rightarrow -1} \frac{5(x + 2)^6}{x^2 + 5x + 6}$

Solution: $\lim_{x \rightarrow -1} \frac{5(x + 2)^6}{x^2 + 5x + 6} = \frac{5(-1 + 2)^6}{(-1)^2 + 5(-1) + 6} = \frac{5}{2}$

**PRACTICE (1)**

Q1: Determine $\lim_{x \rightarrow -5} (-9x^2 - 6x - 9)$.

Q2: Determine $\lim_{x \rightarrow 9} \sqrt{4x^2 - 9x + 1}$.

- ☐ A $2\sqrt{61}$
☐ B 244
☐ C $9\sqrt{3}$
☐ D $11\sqrt{2}$

Q3: Find $\lim_{x \rightarrow -1} (30)$.

Q4: Determine $\lim_{x \rightarrow \frac{\pi}{4}} \frac{9 \sin x}{5x}$.

- ☐ A $\frac{18\sqrt{2}}{5\pi}$
☐ B $\frac{36}{5\pi}$
☐ C $\frac{18\sqrt{2}}{\pi}$
☐ D $\frac{9\sqrt{2}}{10\pi}$
☐ E $\frac{2\sqrt{2}}{5\pi}$

Q5: Assuming that $\lim_{x \rightarrow 6} h(x) = 3$, find $\lim_{x \rightarrow 6} x \cdot h(x)$.

Q6: Given that $\lim_{x \rightarrow -6} \frac{6f(x) + 4}{-x + 1} = 4$, find $\lim_{x \rightarrow -6} f(x)$.

**Example (2)****Common factor****Find the limit of the following functions:**

(1)
$$\lim_{x \rightarrow 0} \frac{x^2 + 6x}{3x}$$

Solution:	$\lim_{x \rightarrow 0} \frac{x^2 + 6x}{3x} = \frac{0}{0}$	(Unspecified quantity)
------------------	--	------------------------

$$\lim_{x \rightarrow 0} \frac{x^2 + 6x}{3x} = \lim_{x \rightarrow 0} \frac{x(x + 6)}{3x} = \lim_{x \rightarrow 0} \frac{x + 6}{3} = \frac{0 + 6}{3} = 2$$

(2)
$$\lim_{x \rightarrow 0} \frac{x^2}{3x^3 - 2x^2}$$

Solution:	$\lim_{x \rightarrow 0} \frac{x^2}{3x^3 - 2x^2} = \frac{0}{0}$	(Unspecified quantity)
------------------	--	------------------------

$$\lim_{x \rightarrow 0} \frac{x^2}{3x^3 - 2x^2} = \lim_{x \rightarrow 0} \frac{x^2}{x^2(3x - 2)} = \lim_{x \rightarrow 0} \frac{1}{3x - 2} = \frac{1}{0 - 2} = -\frac{1}{2}$$

Example (3)**Factorizing Type ①****Find the limit of the following functions:**

(1)
$$\lim_{x \rightarrow -1} \frac{x^2 - 1}{x^2 + x}$$

Solution:	$\lim_{x \rightarrow -1} \frac{x^2 - 1}{x^2 + x} = \frac{0}{0}$	(Unspecified quantity)
------------------	---	------------------------

$$\lim_{x \rightarrow -1} \frac{x^2 - 1}{x^2 + x} = \lim_{x \rightarrow -1} \frac{(x + 1)(x - 1)}{x(x + 1)} = \lim_{x \rightarrow -1} \frac{x - 1}{x} = \frac{-1 - 1}{-1} = 2$$

(2)
$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$$

Solution:	$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} = \frac{0}{0}$	(Unspecified quantity)
------------------	---	------------------------

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} = \lim_{x \rightarrow 5} \frac{(x - 5)(x + 5)}{(x - 5)} = \lim_{x \rightarrow 5} (x + 5) = 5 + 5 = 10$$

**Example (4)****Factorizing Type ②****Find the limit of the following functions:**

$$(1) \quad \lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{5x^2 - 45}$$

Solution: $\lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{5x^2 - 45} = \frac{0}{0}$

(Unspecified quantity)

$$\lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{5x^2 - 45} = \lim_{x \rightarrow 3} \frac{(x-3)(x+5)}{5(x^2-9)} = \lim_{x \rightarrow 3} \frac{(x-3)(x+5)}{5(x-3)(x+3)}$$

$$\lim_{x \rightarrow 3} \frac{(x+5)}{5(x+3)} = \frac{3+5}{5(3+3)} = \frac{4}{15}$$

$$(2) \quad \lim_{x \rightarrow 3} \frac{x^2 - 8x + 15}{x - 3}$$

Solution: $\lim_{x \rightarrow 3} \frac{x^2 - 8x + 15}{x - 3} = \frac{0}{0}$

(Unspecified quantity)

$$\lim_{x \rightarrow 3} \frac{x^2 - 8x + 15}{x - 3} = \lim_{x \rightarrow 3} \frac{(x-3)(x-5)}{(x-3)} = \lim_{x \rightarrow 3} (x-5) = 3-5 = -2$$

$$(3) \quad \lim_{x \rightarrow -2} \frac{x^2 + 7x + 10}{4x + 8}$$

Solution: $\lim_{x \rightarrow -2} \frac{x^2 + 7x + 10}{4x + 8} = \frac{0}{0}$

(Unspecified quantity)

$$\lim_{x \rightarrow -2} \frac{x^2 + 7x + 10}{4x + 8} = \lim_{x \rightarrow -2} \frac{(x+2)(x+5)}{4(x+2)}$$

$$\lim_{x \rightarrow -2} \frac{(x+5)}{4} = \frac{-2+5}{4} = \frac{3}{4}$$

$$(4) \quad \lim_{x \rightarrow -1} \frac{3x^2 + x - 2}{x + 1}$$

Solution: $\lim_{x \rightarrow -1} \frac{3x^2 + x - 2}{x + 1} = \frac{0}{0}$

(Unspecified quantity)

$$\lim_{x \rightarrow -1} \frac{3x^2 + x - 2}{x + 1} = \lim_{x \rightarrow -1} \frac{(x+1)(3x-2)}{(x+1)}$$

$$\lim_{x \rightarrow -1} (3x - 2) = 3(-1) - 2 = -5$$



Example (5)

Factorizing Type ③

Find the limit of the following functions:

(1) $\lim_{x \rightarrow -1} \frac{x^4 + x}{x + 1}$

Solution: $\lim_{x \rightarrow -1} \frac{x^4 + x}{x + 1} = \frac{0}{0}$

(Unspecified quantity)

$$\lim_{x \rightarrow -1} \frac{x^4 + x}{x + 1} = \lim_{x \rightarrow -1} \frac{x(x^3 + 1)}{x + 1} = \lim_{x \rightarrow -1} \frac{x(x + 1)(x^2 - x + 1)}{x + 1}$$

$$\lim_{x \rightarrow -1} x(x^2 - x + 1) = (-1)[(-1)^2 - (-1) + 1] = -3$$

(2) $\lim_{x \rightarrow 4} \frac{x^3 - 64}{x - 4}$

Solution: $\lim_{x \rightarrow 4} \frac{x^3 - 64}{x - 4} = \frac{0}{0}$

(Unspecified quantity)

$$\lim_{x \rightarrow 4} \frac{x^3 - 64}{x - 4} = \lim_{x \rightarrow 4} \frac{(x - 4)(x^2 + 4x + 16)}{(x - 4)}$$

$$\lim_{x \rightarrow 4} (x^2 + 4x + 16) = 4^2 + 4(4) + 16 = 48$$

(3) $\lim_{x \rightarrow -2} \frac{x^4 - 4x^2}{3x^2 - x - 14}$

Solution: $\lim_{x \rightarrow -2} \frac{x^4 - 4x^2}{3x^2 - x - 14} = \frac{0}{0}$

(Unspecified quantity)

$$\lim_{x \rightarrow -2} \frac{x^4 - 4x^2}{3x^2 - x - 14} = \lim_{x \rightarrow -2} \frac{x^2(x^2 - 4)}{(x + 2)(3x - 7)}$$

$$\lim_{x \rightarrow -2} \frac{x^2(x + 2)(x - 2)}{(x + 2)(3x - 7)} = \lim_{x \rightarrow -2} \frac{x^2(x - 2)}{(3x - 7)} = \frac{(-2)^2(-2 - 2)}{3(-2) - 7} = \frac{16}{13}$$

(4) $\lim_{x \rightarrow -3} \frac{(x + 2)^3 + 1}{2x^2 + 6x}$

Solution: $\lim_{x \rightarrow -3} \frac{(x + 2)^3 + 1}{2x^2 + 6x} = \frac{0}{0}$

(Unspecified quantity)

$$\lim_{x \rightarrow -3} \frac{(x + 2)^3 + 1}{2x^2 + 6x} = \lim_{x \rightarrow -3} \frac{[(x + 2) + 1][(x + 2)^2 - (x + 2) + 1]}{2x(x + 3)}$$

$$\lim_{x \rightarrow -3} \frac{(x + 2)^2 - (x + 2) + 1}{2x} = \frac{(-3 + 2)^2 - (-3 + 2) + 1}{2(-3)} = \frac{-1}{2}$$



Examples

Example (1)

Long division

Find the limit of the following functions:

(1) $\lim_{x \rightarrow 1} \frac{x^3 - 2x + 1}{x^2 + x - 2}$

Solution: $\lim_{x \rightarrow 1} \frac{x^3 - 2x + 1}{x^2 + x - 2} = \frac{0}{0}$

(Unspecified quantity)

$$\begin{aligned} & \lim_{x \rightarrow 1} \frac{x^3 - 2x + 1}{x^2 + x - 2} \\ &= \lim_{x \rightarrow 1} \frac{(x-1)(x^2 + x - 1)}{(x-1)(x+2)} \\ &= \lim_{x \rightarrow 1} \frac{(x^2 + x - 1)}{(x+2)} = \frac{1^2 + 1 - 1}{1 + 2} \\ &= \frac{1}{3} \end{aligned}$$

	x^2	$+x$	-1
$x - 1$	x^3		$-2x + 1$
Change sign	x^3	$-x^2$	
	x^2	$-2x$	$+1$
Change sign	x^2	$-x$	
		$-x$	$+1$
Change sign		$-x$	$+1$
		0	0

(2) $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x^3 - 7x - 6}$

Solution: $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x^3 - 7x - 6} = \frac{0}{0}$

(Unspecified quantity)

$$\begin{aligned} & \lim_{x \rightarrow 3} \frac{x^3 - 27}{x^3 - 7x - 6} \\ &= \lim_{x \rightarrow 3} \frac{(x-3)(x^2 + 3x + 9)}{(x-3)(x^2 + 3x + 2)} \\ &= \lim_{x \rightarrow 3} \frac{(x^2 + 3x + 9)}{(x^2 + 3x + 2)} = \frac{3^2 + 3(3) + 9}{3^2 + 3(3) + 2} \\ &= \frac{27}{20} \end{aligned}$$

	x^2	$+3x$	$+2$
$x - 3$	x^3		$-7x - 6$
Change sign	x^3	$-3x^2$	
	$3x^2$	$-7x$	-6
Change sign	$3x^2$	$-9x$	
		$2x$	-6
Change sign		$2x$	-6
		0	0

**Example (2)****Multiply by Conjugate****Find the limit of the following functions:**

$$(1) \quad \lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{x-1}$$

Solution:	$\lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{x-1} = \frac{0}{0}$	(Unspecified quantity)
------------------	---	------------------------

$$\lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{x-1} \times \frac{\sqrt{x+3} + 2}{\sqrt{x+3} + 2} = \lim_{x \rightarrow 1} \frac{(x+3) - 4}{(x-1)(\sqrt{x+3} + 2)}$$

$$\lim_{x \rightarrow 1} \frac{(x-1)}{(x-1)(\sqrt{x+3} + 2)} = \lim_{x \rightarrow 1} \frac{1}{(\sqrt{x+3} + 2)} = \frac{1}{\sqrt{1+3} + 2} = \frac{1}{4}$$

$$(2) \quad \lim_{x \rightarrow 6} \frac{x-6}{\sqrt{x-2} - 2}$$

Solution:	$\lim_{x \rightarrow 6} \frac{x-6}{\sqrt{x-2} - 2} = \frac{0}{0}$	(Unspecified quantity)
------------------	---	------------------------

$$\lim_{x \rightarrow 6} \frac{x-6}{\sqrt{x-2} - 2} \times \frac{\sqrt{x-2} + 2}{\sqrt{x-2} + 2} = \lim_{x \rightarrow 6} \frac{(x-6)(\sqrt{x-2} + 2)}{(x-2) - 4}$$

$$\lim_{x \rightarrow 6} \frac{(x-6)(\sqrt{x-2} + 2)}{(x-6)} = \lim_{x \rightarrow 6} (\sqrt{x-2} + 2) = \sqrt{6-2} + 2 = 4$$

$$(3) \quad \lim_{x \rightarrow 2} \frac{\sqrt{5x-2} - 2\sqrt{2}}{x-2}$$

Solution:	$\lim_{x \rightarrow 2} \frac{\sqrt{5x-2} - 2\sqrt{2}}{x-2} = \frac{0}{0}$	(Unspecified quantity)
------------------	--	------------------------

$$\lim_{x \rightarrow 2} \frac{\sqrt{5x-2} - 2\sqrt{2}}{x-2} \times \frac{\sqrt{5x-2} + 2\sqrt{2}}{\sqrt{5x-2} + 2\sqrt{2}} = \lim_{x \rightarrow 2} \frac{(5x-2) - (2\sqrt{2})^2}{(x-2)(\sqrt{5x-2} + 2\sqrt{2})}$$

$$\lim_{x \rightarrow 2} \frac{5x-10}{(x-2)(\sqrt{5x-2} + 2\sqrt{2})} = \lim_{x \rightarrow 2} \frac{5(x-2)}{(x-2)(\sqrt{5x-2} + 2\sqrt{2})}$$

$$\lim_{x \rightarrow 2} \frac{5}{(\sqrt{5x-2} + 2\sqrt{2})} = \frac{5}{\sqrt{5(2)-2} + 2\sqrt{2}} = \frac{5\sqrt{2}}{8}$$

**Example (3)****By using the theorem****Find the limit of the following functions:**

$$(1) \quad \lim_{x \rightarrow 5} \frac{x^3 - 125}{x - 5}$$

Solution: $\lim_{x \rightarrow 5} \frac{x^3 - 125}{x - 5} = \frac{0}{0}$

(Unspecified quantity)

$$\lim_{x \rightarrow 5} \frac{x^3 - 125}{x - 5} = \lim_{x \rightarrow 5} \frac{x^3 - 5^3}{x - 5} = \frac{3}{1} \times (5)^{3-1} = 75$$

$$(2) \quad \lim_{x \rightarrow 2} \frac{x^5 - 32}{x - 2}$$

Solution: $\lim_{x \rightarrow 2} \frac{x^5 - 32}{x - 2} = \frac{0}{0}$

(Unspecified quantity)

$$\lim_{x \rightarrow 2} \frac{x^5 - 32}{x - 2} = \lim_{x \rightarrow 2} \frac{x^5 - 2^5}{x - 2} = \frac{5}{1} \times (2)^{5-1} = 80$$

$$(3) \quad \lim_{x \rightarrow -3} \frac{x^5 + 243}{x + 3}$$

Solution: $\lim_{x \rightarrow -3} \frac{x^5 + 243}{x + 3} = \frac{0}{0}$

(Unspecified quantity)

$$\lim_{x \rightarrow -3} \frac{x^5 + 243}{x + 3} = \lim_{x \rightarrow -3} \frac{x^5 - (-3)^5}{x - (-3)} = \frac{5}{1} \times (-3)^{5-1} = 405$$

$$(4) \quad \lim_{x \rightarrow \sqrt{2}} \frac{x^8 - 16}{x - \sqrt{2}}$$

Solution: $\lim_{x \rightarrow \sqrt{2}} \frac{x^8 - 16}{x - \sqrt{2}} = \frac{0}{0}$

(Unspecified quantity)

$$\lim_{x \rightarrow \sqrt{2}} \frac{x^8 - 16}{x - \sqrt{2}} = \lim_{x \rightarrow \sqrt{2}} \frac{x^8 - (\sqrt{2})^8}{x - \sqrt{2}} = \frac{8}{1} \times (\sqrt{2})^{8-1} = 64\sqrt{2}$$



(5) $\lim_{x \rightarrow -1} \frac{x^{15} + x^{14}}{x^4 - 1}$

Solution: $\lim_{x \rightarrow -1} \frac{x^{15} + x^{14}}{x^4 - 1} = \frac{0}{0}$

(Unspecified quantity)

$$\begin{aligned} \lim_{x \rightarrow -1} \frac{x^{15} + x^{14}}{x^4 - 1} &= \lim_{x \rightarrow -1} \frac{x^{14}(x + 1)}{x^4 - 1^4} \\ &= \lim_{x \rightarrow -1} x^{14} \times \lim_{x \rightarrow -1} \frac{x - (-1)}{x^4 - (-1)^4} = (-1)^{14} \times \frac{1}{4} \times (-1)^{1-4} = \frac{-1}{4} \end{aligned}$$

(6) $\lim_{x \rightarrow -2} \frac{x^4 - 16}{x^5 + 32}$

Solution: $\lim_{x \rightarrow -2} \frac{x^4 - 16}{x^5 + 32} = \frac{0}{0}$

(Unspecified quantity)

$$\lim_{x \rightarrow -2} \frac{x^4 - 16}{x^5 + 32} = \lim_{x \rightarrow -2} \frac{x^4 - 2^4}{x^5 + 2^5} = \lim_{x \rightarrow -2} \frac{x^4 - (-2)^4}{x^5 - (-2)^5} = \frac{4}{5} \times (-2)^{4-5} = \frac{-2}{5}$$

(7) $\lim_{x \rightarrow 0} \frac{(x + 2)^5 - 32}{x}$

Solution: $\lim_{x \rightarrow 0} \frac{(x + 2)^5 - 32}{x} = \frac{0}{0}$

(Unspecified quantity)

$$\lim_{x \rightarrow 0} \frac{(x + 2)^5 - 32}{x} = \lim_{(x+2) \rightarrow 2} \frac{(x + 2)^5 - 2^5}{(x + 2) - 2} = \frac{5}{1} \times (2)^{5-1} = 80$$

(8) $\lim_{h \rightarrow 0} \frac{(1 + 4h)^8 - 1}{h}$

Solution: $\lim_{h \rightarrow 0} \frac{(1 + 4h)^8 - 1}{h} = \frac{0}{0}$

(Unspecified quantity)

$$\lim_{h \rightarrow 0} \frac{(1 + 4h)^8 - 1}{h} = 4 \times \lim_{(1+4h) \rightarrow 1} \frac{(1 + 4h)^8 - 1^8}{(1 + 4h) - 1}$$

$$\lim_{h \rightarrow 0} \frac{(1 + 4h)^8 - 1}{h} = 4 \times \frac{8}{1} \times (1)^{8-1} = 32$$

(9) $\lim_{x \rightarrow 3} \frac{(x - 2)^9 - 1}{x - 3}$

Solution: $\lim_{x \rightarrow 3} \frac{(x - 2)^9 - 1}{x - 3} = \frac{0}{0}$

(Unspecified quantity)

$$\lim_{x \rightarrow 0} \frac{(1 + 2x)^9 - 1}{5x} = \frac{2}{5} \times \lim_{(1+2x) \rightarrow 1} \frac{(1 + 2x)^9 - 1^9}{(1 + 2x) - 1} = \frac{2}{5} \times \frac{9}{1} \times (1)^{9-1} = \frac{18}{5}$$

**PRACTICE (2)**

Q1: Find $\lim_{x \rightarrow 4} \frac{3x^2 - 18x + 24}{x^2 - 16}$.

- ☐ A $\frac{3}{4}$
- ☐ B 0
- ☐ C 4
- ☐ D $\frac{3}{2}$
- ☐ E $\frac{9}{4}$

Q2: Determine $\lim_{x \rightarrow 2} \frac{x^5 - 32}{x - 2}$.

Q3: Find $\lim_{x \rightarrow 2} \frac{x^2 - 2x}{2x^2 - 6x + 4}$.

- ☐ A 1
- ☐ B -1
- ☐ C 2
- ☐ D 0
- ☐ E $\frac{1}{3}$

Q4: Find $\lim_{x \rightarrow -2} \frac{x^8 - 256}{x^4 - 16}$.

- ☐ A 32
- ☐ B The limit does not exist.
- ☐ C $\frac{1}{8}$
- ☐ D 8



Q5: Find $\lim_{x \rightarrow 2} \frac{8x^3 - 64}{x^2 - 4}$.

Q6: Find $\lim_{x \rightarrow 0} \frac{(x - 4)^2 - 16}{x}$.

Q7: Find $\lim_{x \rightarrow 2} \frac{(x^2 - 4)^2}{4x - 8}$.

Q8: Find $\lim_{x \rightarrow -1} \frac{(x - 6)(x^2 + 2x + 1)}{x^2 - 6x - 7}$.

Q9: Find $\lim_{3x \rightarrow 1} \frac{9x^2 - 1}{27x^3 - 1}$.

A $\frac{2}{3}$

B 2

C The limit doesn't exist.

D $\frac{3}{2}$

Q10: Find $\lim_{x \rightarrow 8} \frac{6x - 48}{6x + (x - 8)^8 - 48}$.

Q11: Find $\lim_{x \rightarrow -3} \frac{x^2 - 9}{x + 3}$.

Q12: Find $\lim_{x \rightarrow -\frac{3}{5}} \frac{-25x^2 - 35x - 12}{5x + 3}$.



Q13: Find $\lim_{x \rightarrow -4} \frac{x^3 + 64}{2x^2 + 6x - 8}$.

- ☐ A 0
- ☐ B $-\frac{24}{5}$
- ☐ C $-\frac{8}{5}$
- ☐ D 48

Q14: Find $\lim_{x \rightarrow -6} \frac{x^2 - 36}{x^3 + 216}$.

- ☐ A -9
- ☐ B $\frac{2}{3}$
- ☐ C $-\frac{1}{9}$
- ☐ D The limit does not exist.
- ☐ E $\frac{3}{2}$

Q15: Find $\lim_{x \rightarrow -8} \frac{6x^3 + 53x^2 + 43x + 24}{3x^2 + 22x - 16}$.

- ☐ A $-\frac{347}{8}$
- ☐ B 347
- ☐ C The limit does not exist.
- ☐ D $\frac{347}{26}$
- ☐ E $-\frac{347}{26}$



Exercises 3 - 2



Complete:

- ① $\lim_{x \rightarrow 2} (3x - 1) = \dots\dots\dots$ ② $\lim_{x \rightarrow 1} \frac{x - 3}{x + 1} = \dots\dots\dots$ ③ $\lim_{x \rightarrow 0} \frac{x^2 - x}{x} = \dots\dots\dots$
- ④ $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \dots\dots\dots$ ⑤ $\lim_{x \rightarrow a} \frac{x^5 - a^5}{x - a} = \dots\dots\dots$ ⑥ $\lim_{x \rightarrow 2} \frac{\sqrt[3]{x} - \sqrt[3]{2}}{x - 2} = \dots\dots\dots$
- ⑦ $\lim_{x \rightarrow 1} \frac{\frac{1}{x^3} - 1}{x^4 - 1} = \dots\dots\dots$ ⑧ $\lim_{x \rightarrow 2} \frac{x^{-1} - 2^{-1}}{x^3 - 2^3} = \dots\dots\dots$

Choose the correct answer from those given:

- ⑨ $\lim_{x \rightarrow -1} \frac{x^3 - 1}{x + 1}$ equals:
 a -3 b -2 c 3 d has no limit
- ⑩ $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x}{x}$ equals:
 a 1 b $\frac{\pi}{2}$ c $\frac{2}{\pi}$ d has no limit
- ⑪ $\lim_{x \rightarrow 16} \frac{\sqrt{x} - 1}{x - 16}$ equals:
 a zero b $\frac{1}{2}$ c 1 d has no limit
- ⑫ $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x}{x}$ equals:
 a 0 b 1 c $\frac{4}{\pi}$ d has no limit
- ⑬ $\lim_{x \rightarrow 3} \frac{x^5 - 243}{x^3 - 27}$ equals:
 a 0 b $\frac{5}{3}$ c 15 d 9
- ⑭ $\lim_{x \rightarrow 2} \frac{x^2 - 4a}{x - 2}$ exists then a equals:
 a -1 b 1 c 2 d 4
- ⑮ $\lim_{x \rightarrow 2} \frac{5}{(x-2)^3}$ equals:
 a $-\frac{5}{2}$ b zero c 5 d has no limit



Find each of the following limits (if exist):

$$16 \quad \lim_{x \rightarrow 3} (x^2 - 3x + 2)$$

$$17 \quad \lim_{x \rightarrow -2} \frac{x^2 + 1}{x - 3}$$

$$18 \quad \lim_{x \rightarrow \frac{\pi}{2}} (2x - \sin x)$$

$$19 \quad \lim_{x \rightarrow \pi} \frac{\cos 2x}{x}$$

$$20 \quad \lim_{x \rightarrow -1} \frac{x + 1}{x^3 + 1}$$

$$21 \quad \lim_{x \rightarrow 9} \frac{9 - x}{x^2 - 81}$$

$$22 \quad \lim_{x \rightarrow 2} \frac{x^2 - 7x + 10}{x^2 - 2x}$$

$$23 \quad \lim_{x \rightarrow -1} \frac{x^2 + 6x + 5}{x^2 - 3x - 4}$$

$$24 \quad \lim_{x \rightarrow 9} \frac{x + \sqrt{x} - 12}{x - 9}$$

$$25 \quad \lim_{x \rightarrow 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x}$$

$$26 \quad \lim_{x \rightarrow -2} \frac{x + 2}{x^4 - 16}$$

$$27 \quad \lim_{x \rightarrow 1} \left(\frac{2}{x} + \frac{x^2 - x}{x - 1} \right)$$

$$28 \quad \lim_{x \rightarrow 2} \frac{x^3 + 3x^2 - 12x + 4}{x^3 - 4x}$$

$$29 \quad \lim_{x \rightarrow 4} \frac{x^3 - 4x^2 - x + 4}{2x^2 - 7x - 4}$$

$$30 \quad \lim_{x \rightarrow 3} \frac{x^2 - 9}{x^3 - 2x^2 + 2x - 15}$$

$$31 \quad \lim_{x \rightarrow 2} \frac{x^5 - 32}{x - 2}$$

$$32 \quad \lim_{x \rightarrow 2} \frac{x^{-8} - (16)^{-2}}{x - 2}$$

$$33 \quad \lim_{x \rightarrow 1} \frac{\sqrt[7]{x} - 1}{x - 1}$$

$$34 \quad \lim_{x \rightarrow 2} \frac{x^9 - \frac{1}{512}}{x - 2}$$

$$35 \quad \lim_{x \rightarrow \sqrt{5}} \frac{x^7 - 125\sqrt{5}}{x^4 - \sqrt{25}}$$

$$36 \quad \lim_{x \rightarrow 81} \frac{\sqrt[4]{x} - 3}{x - 81}$$

$$37 \quad \lim_{x \rightarrow 2} \frac{x^7 - 128}{x^5 - 32}$$

$$38 \quad \lim_{x \rightarrow 6} \frac{(x - 5)^7 - 1}{x - 6}$$

$$39 \quad \lim_{x \rightarrow 7} \frac{\sqrt[5]{x + 25} - 2}{x - 7}$$

$$40 \quad \lim_{x \rightarrow 2} \frac{(x - 3)^6 - 1}{x - 2}$$

$$41 \quad \lim_{h \rightarrow 0} \frac{(x - 2h)^{17} - x^{17}}{51h}$$

$$42 \quad \lim_{x \rightarrow -3} \frac{x^4 - 81}{x^5 + 243}$$

$$43 \quad \lim_{h \rightarrow 0} \frac{(3 + h)^4 - 81}{6h}$$

$$44 \quad \lim_{x \rightarrow -1} \frac{\sqrt{x^2 + 8} - 3}{x + 1}$$

$$45 \quad \lim_{x \rightarrow -3} \frac{\sqrt{x + 7} - 2}{x + 3}$$

$$46 \quad \lim_{x \rightarrow 3} \frac{\sqrt{x + 1} - 2}{\sqrt{x + 6} - 3}$$

$$47 \quad \lim_{x \rightarrow -1} \frac{x^2 - x - 2}{x + 1}$$

$$48 \quad \lim_{x \rightarrow 0} \frac{(2x - 1)^2 - 1}{5x}$$

$$49 \quad \lim_{x \rightarrow 1} \left(\frac{1}{x - 1} - \frac{3}{x^3 - 1} \right)$$

$$50 \quad \lim_{x \rightarrow -2} \frac{x^2 + 4x + 4}{x^3 + x^2 - 8x - 12}$$

$$51 \quad \lim_{x \rightarrow 4} \frac{\sqrt{2x + 1} - 3}{x - 4}$$

**Lesson (3)****Limits Of The Function At Infinity****Lesson objectives****Related Links**

-  Determine Limit of a function at infinity
-  Find the limit of a function at infinity algebraically.
-  Find the limit of a function at infinity graphically.

Important Notes:

- ① $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$
- ② $\lim_{x \rightarrow \infty} \frac{a}{x} = 0 \quad \forall a \in \mathbb{R}^*$
- ③ $\lim_{x \rightarrow \infty} \frac{a}{x^n} = 0 \quad \forall a \in \mathbb{R}^* \text{ and } n \in \mathbb{R}^+$
- ④ If the direct substituting gives $\frac{\infty}{\infty}$, then we divide each of the numerator & the denominator by x raised to the higher power in the denominator.

Important Rules:

$$\lim_{x \rightarrow \infty} c = c$$

$$\lim_{x \rightarrow \infty} x^n = \infty$$



Examples

Example (1)

Find the limit of the following functions:

(1) $\lim_{x \rightarrow \infty} \frac{4 - 2x}{3x - 1}$

Solution: $\lim_{x \rightarrow \infty} \frac{4 - 2x}{3x - 1} = \frac{4 - \infty}{\infty - 1} = \frac{\infty}{\infty}$

$$\lim_{x \rightarrow \infty} \frac{4 - 2x}{3x - 1} \quad \div \frac{x}{x}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{4}{x} - 2}{3 - \frac{1}{x}} = \frac{0 - 2}{3 - 0} = \frac{-2}{3}$$

(2) $\lim_{x \rightarrow \infty} \frac{2x^2 - 7}{5x^2 + 4x + 8}$

Solution: $\lim_{x \rightarrow \infty} \frac{2x^2 - 7}{5x^2 + 4x + 8} = \frac{\infty - 7}{\infty + \infty + 8} = \frac{\infty}{\infty}$

$$\lim_{x \rightarrow \infty} \frac{2x^2 - 7}{5x^2 + 4x + 8} \quad \div \frac{x^2}{x^2}$$

$$\lim_{x \rightarrow \infty} \frac{2 - \frac{7}{x^2}}{5 + \frac{4}{x} + \frac{8}{x^2}} = \frac{2 - 0}{5 + 0 + 0} = \frac{2}{5}$$

(3) $\lim_{x \rightarrow \infty} \frac{x^4 + 4}{2x^3 + 9}$

Solution: $\lim_{x \rightarrow \infty} \frac{x^4 + 4}{2x^3 + 9} = \frac{\infty + 4}{\infty + 9} = \frac{\infty}{\infty}$

$$\lim_{x \rightarrow \infty} \frac{x^4 + 4}{2x^3 + 9} \quad \div \frac{x^3}{x^3}$$

$$\lim_{x \rightarrow \infty} \frac{x + \frac{4}{x^3}}{2 + \frac{9}{x^3}} = \frac{\infty + 0}{2 + 0} = \infty$$



Example (2)

Find the limit of the following functions:

$$(1) \quad \lim_{x \rightarrow \infty} \frac{(2x + 3)^2}{5 - 3x - x^2}$$

Solution: $\lim_{x \rightarrow \infty} \frac{(2x + 3)^2}{5 - 3x - x^2} = \frac{(\infty + 3)^2}{5 - \infty - \infty} = \frac{\infty}{\infty}$

$$\lim_{x \rightarrow \infty} \frac{(2x + 3)^2}{5 - 3x - x^2} \quad \div \frac{x^2}{x^2}$$

$$\lim_{x \rightarrow \infty} \frac{\left(2 + \frac{3}{x}\right)^2}{\frac{5}{x^2} - \frac{3}{x} - 1} = \frac{(2 + 0)^2}{0 - 0 - 1} = -4$$

$$(2) \quad \lim_{x \rightarrow \infty} \frac{5 - 4x + 3x^2}{(x + 2)^2}$$

Solution: $\lim_{x \rightarrow \infty} \frac{5 - 4x + 3x^2}{(x + 2)^2} = \frac{5 - \infty + \infty}{(\infty + 2)^2} = \frac{\infty}{\infty}$

$$\lim_{x \rightarrow \infty} \frac{5 - 4x + 3x^2}{(x + 2)^2} \quad \div \frac{x^2}{x^2}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{5}{x^2} - \frac{4}{x} + 3}{\left(1 + \frac{2}{x}\right)^2} = \frac{0 - 0 + 3}{(1 + 0)^2} = 3$$

$$(3) \quad \lim_{x \rightarrow \infty} \frac{8x^3 - x + 1}{(x + 1)(2x^2 - 3)}$$

Solution: $\lim_{x \rightarrow \infty} \frac{8x^3 - x + 1}{(x + 1)(2x^2 - 3)} = \frac{\infty - \infty + 1}{(\infty + 1)(\infty - 3)} = \frac{\infty}{\infty}$

$$\lim_{x \rightarrow \infty} \frac{8x^3 - x + 1}{(x + 1)(2x^2 - 3)} \quad \div \frac{x^3}{x^3}$$

$$\lim_{x \rightarrow \infty} \frac{8 - \frac{1}{x^2} + \frac{1}{x^3}}{\left(1 + \frac{1}{x}\right)\left(2 - \frac{3}{x^2}\right)} = \frac{8 - 0 + 0}{(1 + 0)(2 - 0)} = 4$$



Example (3)

Find the limit of the following functions:

$$(1) \quad \lim_{x \rightarrow \infty} \frac{5x - 20}{\sqrt[3]{27x^3 + 8}}$$

Solution: $\lim_{x \rightarrow \infty} \frac{5x - 20}{\sqrt[3]{27x^3 + 8}} = \frac{\infty - \infty}{\sqrt[3]{\infty + 8}} = \frac{\infty}{\infty}$

$$\lim_{x \rightarrow \infty} \frac{5x - 20}{\sqrt[3]{27x^3 + 8}} \quad \div \frac{x}{x}$$

$$\lim_{x \rightarrow \infty} \frac{5 - \frac{20}{x}}{\sqrt[3]{27 + \frac{8}{x^3}}} = \frac{5 - 0}{\sqrt[3]{27 + 0}} = \frac{5}{3}$$

$$(2) \quad \lim_{x \rightarrow \infty} \frac{2x + 1}{\sqrt{4x^2 + 3x - 4}}$$

Solution: $\lim_{x \rightarrow \infty} \frac{2x + 1}{\sqrt{4x^2 + 3x - 4}} = \frac{\infty + 1}{\sqrt{\infty + \infty - 4}} = \frac{\infty}{\infty}$

$$\lim_{x \rightarrow \infty} \frac{2x + 1}{\sqrt{4x^2 + 3x - 4}} \quad \div \frac{x}{x}$$

$$\lim_{x \rightarrow \infty} \frac{2 + \frac{1}{x}}{\sqrt{4 + \frac{3}{x} - \frac{4}{x^2}}} = \frac{2 + 0}{\sqrt{4 + 0 - 0}} = 1$$

$$(3) \quad \lim_{x \rightarrow \infty} \frac{6x + 5}{\sqrt{81x^2 - 1}}$$

Solution: $\lim_{x \rightarrow \infty} \frac{6x + 5}{\sqrt{81x^2 - 1}} = \frac{\infty + 5}{\sqrt{\infty - 1}} = \frac{\infty}{\infty}$

$$\lim_{x \rightarrow \infty} \frac{6x + 5}{\sqrt{81x^2 - 1}} \quad \div \frac{x}{x}$$

$$\lim_{x \rightarrow \infty} \frac{6 + \frac{5}{x}}{\sqrt{81 - \frac{1}{x^2}}} = \frac{6 + 0}{\sqrt{81 - 0}} = \frac{2}{3}$$



Example (4)

Find the limit of the following functions:

(1) $\lim_{x \rightarrow \infty} \frac{x^3 - 2}{|x|^3 + 1}$

Solution: $\lim_{x \rightarrow \infty} \frac{x^3 - 2}{|x|^3 + 1} = \frac{\infty - 2}{\infty + 1} = \frac{\infty}{\infty}$

$$\because x \rightarrow 0$$

$$\because x > 0$$

$$\text{But } |x| > 0$$

$$\lim_{x \rightarrow \infty} \frac{x^3 - 2}{x^3 + 1}$$

$$\div \frac{x^3}{x^3}$$

$$\lim_{x \rightarrow \infty} \frac{1 - \frac{2}{x^3}}{1 + \frac{1}{x^3}} = \frac{1 - 0}{1 + 0} = 1$$

(2) $\lim_{x \rightarrow \infty} (x - \sqrt{x^2 + 4})$

Solution: $\lim_{x \rightarrow \infty} (x - \sqrt{x^2 + 4}) = \infty - \sqrt{\infty + 4} = \infty - \infty$ (unspecified quantity)

$$\lim_{x \rightarrow \infty} \frac{(x - \sqrt{x^2 + 4})}{1}$$

$$\times \frac{(x + \sqrt{x^2 + 4})}{(x + \sqrt{x^2 + 4})}$$

$$\lim_{x \rightarrow \infty} \frac{(x - \sqrt{x^2 + 4}) \times (x + \sqrt{x^2 + 4})}{(x + \sqrt{x^2 + 4})}$$

$$\lim_{x \rightarrow \infty} \frac{x^2 - (x^2 + 4)}{(x + \sqrt{x^2 + 4})}$$

$$\lim_{x \rightarrow \infty} \frac{-4}{x + \sqrt{x^2 + 4}}$$

$$\div \frac{x}{x}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{-4}{x}}{1 + \sqrt{1 + \frac{4}{x^2}}} = \frac{0}{1 + \sqrt{1 + 0}} = 0$$

**PRACTICE**

Q1: Find $\lim_{x \rightarrow \infty} \frac{6x^2}{x - 6}$.

- ☐ A -1
- ☐ B ∞
- ☐ C 6
- ☐ D 0

Q2: Find $\lim_{x \rightarrow \infty} \sqrt{\frac{16x + 8}{9x + 3}}$.

- ☐ A ∞
- ☐ B $\frac{4}{3}$
- ☐ C $\frac{16}{9}$
- ☐ D 0

Q3: Find $\lim_{x \rightarrow \infty} \frac{x^2 + 3}{8x^3 + 9x + 1}$.

- ☐ A ∞
- ☐ B $\frac{1}{8}$
- ☐ C 3
- ☐ D 0



Q4: Find $\lim_{x \rightarrow \infty} \left(\frac{3x}{8x+4} - \frac{7x^2}{(2x+7)^2} \right)$.

- ☐ A $-\frac{11}{8}$
- ☐ B ∞
- ☐ C $\frac{45}{196}$
- ☐ D 0

Q5: Find $\lim_{x \rightarrow \infty} -\frac{3x^2}{-4x^2+8}$.

- ☐ A 0
- ☐ B $-\frac{3}{8}$
- ☐ C ∞
- ☐ D $\frac{3}{4}$

Q6: Find $\lim_{x \rightarrow \infty} \frac{3x^4 - x^3 - x^2 + 3x + 2}{x^4 - 8x^3 - x^2 - 5x - 8}$.

- ☐ A ∞
- ☐ B $-\infty$
- ☐ C -3
- ☐ D 3

Q7: Find $\lim_{x \rightarrow -\infty} 9 - 8x + 6x^2 - 2x^3$.

- ☐ A 5
- ☐ B 9
- ☐ C ∞
- ☐ D $-\infty$
- ☐ E -2



Exercises 3 - 3



Complete the following :

① $\lim_{x \rightarrow \infty} (1 + \frac{3}{x}) = \dots\dots\dots$

② $\lim_{x \rightarrow \infty} (\frac{3}{x^2} - 2) = \dots\dots\dots$

③ $\lim_{x \rightarrow \infty} (-7) = \dots\dots\dots$

④ $\lim_{x \rightarrow \infty} (x^2 - 3) = \dots\dots\dots$

⑤ $\lim_{x \rightarrow \infty} \frac{2x+1}{x} = \dots\dots\dots$

⑥ $\lim_{x \rightarrow \infty} \frac{x^3 - 5}{x^2 + 1} = \dots\dots\dots$

⑦ $\lim_{x \rightarrow \infty} \frac{x^5 + 3}{x^3 - 5} = \dots\dots\dots$

⑧ $\lim_{x \rightarrow \infty} \frac{3x}{\sqrt{x^2 - 1}} = \dots\dots\dots$

⑨ $\lim_{x \rightarrow \infty} (3 - \frac{7}{x} + \frac{4}{x^2}) = \dots\dots\dots$

⑩ $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - x) = \dots\dots\dots$

Choose the correct answer from those given:

⑪ $\lim_{x \rightarrow \infty} \frac{6x}{2x+3}$ equals

- a** 0 **b** 2 **c** 3 **d** $+\infty$

⑫ $\lim_{x \rightarrow \infty} \sqrt{\frac{4}{x} + 1}$ equals

- a** 0 **b** 1 **c** 2 **d** $+\infty$

⑬ $\lim_{x \rightarrow \infty} \frac{x+3}{2-x^2}$ equals

- a** 0 **b** $\frac{1}{2}$ **c** $\frac{3}{2}$ **d** $+\infty$

⑭ $\lim_{x \rightarrow \infty} \frac{x^2 + 1}{2x - 1}$ equals

- a** 0 **b** $\frac{1}{2}$ **c** 1 **d** $+\infty$

⑮ $\lim_{x \rightarrow \infty} \sqrt{\frac{1+x}{4x-1}}$ equals

- a** -1 **b** $\frac{1}{4}$ **c** $\frac{1}{2}$ **d** 1



Find

$$\textcircled{16} \quad \lim_{x \rightarrow \infty} \frac{3}{x^2}$$

$$\textcircled{17} \quad \lim_{x \rightarrow \infty} (x^3 + 5x^2 + 1)$$

$$\textcircled{18} \quad \lim_{x \rightarrow \infty} \frac{2 - 7x}{2 + 3x}$$

$$\textcircled{19} \quad \lim_{x \rightarrow \infty} \frac{x^2}{x + 3}$$

$$\textcircled{20} \quad \lim_{x \rightarrow \infty} \frac{4x^2}{x^2 + 3}$$

$$\textcircled{21} \quad \lim_{x \rightarrow \infty} \frac{5 - 6x - 3x}{2x^2 + x + 4}$$

$$\textcircled{22} \quad \lim_{x \rightarrow \infty} \frac{2x - 1}{x^2 + 4x + 1}$$

$$\textcircled{23} \quad \lim_{x \rightarrow \infty} \frac{x^3 - 2}{3x^2 + 4x - 1}$$

$$\textcircled{24} \quad \lim_{x \rightarrow \infty} \frac{2x^2 - 1}{4x^3 - 5x - 1}$$

$$\textcircled{25} \quad \lim_{x \rightarrow \infty} \frac{2x^2 - 6}{(x - 1)^2}$$

$$\textcircled{26} \quad \lim_{x \rightarrow \infty} \left(7 + \frac{2x^2}{(x + 3)^2} \right)$$

$$\textcircled{27} \quad \lim_{x \rightarrow \infty} \left(\frac{1}{3x^2} - \frac{5x}{2 + x} \right)$$

$$\textcircled{28} \quad \lim_{x \rightarrow \infty} \left(\frac{x}{2x + 1} + \frac{3x^2}{(x - 3)^2} \right)$$

$$\textcircled{29} \quad \lim_{x \rightarrow \infty} \frac{-x}{\sqrt{4 + x^2}}$$

$$\textcircled{30} \quad \lim_{x \rightarrow \infty} \frac{x^3 - 4x + 5}{(2x - 1)^3}$$

$$\textcircled{31} \quad \lim_{x \rightarrow \infty} (\sqrt{4x^2 - 2x + 1} - 2x)$$

$$\textcircled{32} \quad \lim_{x \rightarrow \infty} (\sqrt{5x^2 + 4x + 7} - \sqrt{5x^2 + x + 3})$$

$$\textcircled{33} \quad \lim_{x \rightarrow \infty} x(\sqrt{4x^2 + 1} - 2x)$$

$$\textcircled{34} \quad \lim_{x \rightarrow \infty} \frac{x^2 + x - 1}{8x^2 - 3}$$

$$\textcircled{35} \quad \lim_{x \rightarrow \infty} \frac{4 - 3x^3}{\sqrt{x^6 + 9}}$$

$$\textcircled{36} \quad \lim_{x \rightarrow \infty} \frac{(x + 2)^3(3 - 2x^2)}{3x(x^2 + 7)^2}$$

$$\textcircled{37} \quad \text{If } \lim_{x \rightarrow \infty} (\sqrt{ax^2 + 3bx + 5} - 2x) = 3 \text{ find the value of each of a, b.}$$

$$\textcircled{38} \quad \lim_{x \rightarrow \infty} \frac{x^{-1} + 3x^{-2} + 5}{2x^{-2} - x^{-3} + 1}$$

$$\textcircled{39} \quad \lim_{x \rightarrow \infty} \frac{2x^{-1} - 3x^{-2} + x^{-3}}{4x^{-3} + x^{-1}}$$

**Lesson (4)****Limits of Trigonometric Functions****Lesson objectives****Related Links**

- ✚ Solve the limit of the sine function.
- ✚ Solve the limit of the tangent function.

**Important Rules:**

- ① $\lim_{x \rightarrow a} \sin x = \sin a$
- ② $\lim_{x \rightarrow a} \cos x = \cos a$
- ③ $\lim_{x \rightarrow a} \tan x = \tan a : a \neq \frac{2n+1}{2}\pi, \quad n \in \mathbb{Z}$
- ④ $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \& \quad \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$
- ⑤ $\lim_{x \rightarrow 0} \frac{\sin ax}{bx} = \frac{a}{b} \quad \& \quad \lim_{x \rightarrow 0} \frac{\tan ax}{bx} = \frac{a}{b}$

🕒 Observe the difference between:

~~$\lim_{x \rightarrow 0} \frac{\sin ax^2}{x^2} = a$~~

~~$\lim_{x \rightarrow 0} \frac{\sin^2 ax}{x^2} = \lim_{x \rightarrow 0} \left(\frac{\sin ax}{x} \right)^2 = a^2$~~



Example (1)

Find the limit of the following functions:

(1) $\lim_{x \rightarrow 0} \frac{\sin 2x}{x}$

Solution: $\lim_{x \rightarrow 0} \frac{\sin 2x}{x} = 2$

(2) $\lim_{x \rightarrow 0} \frac{\sin 3x}{\tan 7x}$

Solution: $\lim_{x \rightarrow 0} \frac{\sin 3x}{\tan 7x} \div \frac{x}{x}$

$$\lim_{x \rightarrow 0} \frac{\frac{\sin 3x}{x}}{\frac{\tan 7x}{x}} = \frac{3}{7}$$

(3) $\lim_{x \rightarrow 0} \frac{\sin \pi x}{6x}$

Solution: $\lim_{x \rightarrow 0} \frac{\sin \pi x}{6x} \div \frac{x}{x}$

$$\lim_{x \rightarrow 0} \frac{\frac{\sin \pi x}{x}}{6} = \frac{\pi}{6}$$

(4) $\lim_{x \rightarrow 0} \frac{x \cos 3x}{\tan 2x}$

Solution: $\lim_{x \rightarrow 0} \frac{x \cos 3x}{\tan 2x} \div \frac{x}{x}$

$$\lim_{x \rightarrow 0} \frac{\cos 3x}{\frac{\tan 2x}{x}} = \frac{1}{2}$$

(5) $\lim_{2x \rightarrow 3} \frac{\tan(2x - 3)}{2x - 3}$

Solution: $\lim_{2x \rightarrow 3} \frac{\tan(2x - 3)}{2x - 3} = \lim_{(2x-3) \rightarrow 0} \frac{\tan(2x - 3)}{(2x - 3)} = 1$

(6) $\lim_{x \rightarrow 0} \frac{4 + 5x}{\cos 3x}$

Solution: $\lim_{x \rightarrow 0} \frac{4 + 5x}{\cos 3x} = \frac{4 + 0}{1} = 4$



Example (2)

Find the limit of the following functions:

$$(1) \quad \lim_{x \rightarrow 0} \frac{\sin x + \tan x}{\sin x}$$

Solution:

$$\lim_{x \rightarrow 0} \frac{\sin x + \tan x}{\sin x} \quad \div \frac{x}{x}$$

$$\lim_{x \rightarrow 0} \frac{\frac{\sin x}{x} + \frac{\tan x}{x}}{\frac{\sin x}{x}} = \frac{1 + 1}{1} = 2$$

$$(2) \quad \lim_{x \rightarrow 0} \frac{\tan 3x + 2x \cos x}{\sin 5x}$$

Solution:

$$\lim_{x \rightarrow 0} \frac{\tan 3x + 2x \cos x}{\sin 5x} \quad \div \frac{x}{x}$$

$$\lim_{x \rightarrow 0} \frac{\frac{\tan 3x}{x} + 2 \cos x}{\frac{\sin 5x}{x}} = \frac{3 + 2}{5} = 1$$

$$(3) \quad \lim_{x \rightarrow 0} \frac{\sin 2x + 4x}{3x - \tan x}$$

Solution:

$$\lim_{x \rightarrow 0} \frac{\sin 2x + 4x}{3x - \tan x} \quad \div \frac{x}{x}$$

$$\lim_{x \rightarrow 0} \frac{\frac{\sin 2x}{x} + 4}{3 - \frac{\tan x}{x}} = \frac{2 + 4}{3 - 1} = 3$$

$$(4) \quad \lim_{x \rightarrow 0} \frac{2x + 5 \tan x + 3 \sin x}{\sin 2x \times \cos 2x}$$

Solution:

$$\lim_{x \rightarrow 0} \frac{2x + 5 \tan x + 3 \sin x}{\sin 2x \times \cos 2x} \quad \div \frac{x}{x}$$

$$\lim_{x \rightarrow 0} \frac{2 + \frac{5 \tan x}{x} + \frac{3 \sin x}{x}}{\frac{\sin 2x}{x} \times \cos 2x} = \frac{2 + 5 + 3}{2} = 5$$

$$(5) \quad \lim_{x \rightarrow 0} \frac{2 \sin x - x \cos x}{\sin x + x \cos x}$$

Solution:

$$\lim_{x \rightarrow 0} \frac{2 \sin x - x \cos x}{\sin x + x \cos x} \quad \div \frac{x}{x}$$

$$\lim_{x \rightarrow 0} \frac{\frac{2 \sin x}{x} - \cos x}{\frac{\sin x}{x} + \cos x} = \frac{2 - 1}{1 + 1} = \frac{1}{2}$$



Example (3)

Find the limit of the following functions:

(1) $\lim_{x \rightarrow 0} \frac{\sin 3x^2}{x^2}$

Solution: $\lim_{x \rightarrow 0} \frac{\sin^2 3x^2}{x^2} = 3$

(2) $\lim_{x \rightarrow 0} \frac{\sin^2 5x}{2x^2}$

Solution: $\lim_{x \rightarrow 0} \frac{\sin^2 5x}{2x^2} = \frac{1}{2} \lim_{x \rightarrow 0} \frac{\sin^2 5x}{x^2} = \frac{1}{2} \lim_{x \rightarrow 0} \left(\frac{\sin 5x}{x} \right)^2 = \frac{1}{2} \times 5^2 = \frac{25}{2}$

(3) $\lim_{x \rightarrow 0} \frac{\tan^3 2x}{4x^3}$

Solution: $\lim_{x \rightarrow 0} \frac{\tan^3 2x}{4x^3} = \frac{1}{4} \lim_{x \rightarrow 0} \frac{\tan^3 2x}{x^3} = \frac{1}{4} \lim_{x \rightarrow 0} \left(\frac{\tan 2x}{x} \right)^3 = \frac{1}{4} \times 2^3 = 2$

(4) $\lim_{x \rightarrow 0} \frac{\tan^2 3x}{x \sin 2x}$

Solution: $\lim_{x \rightarrow 0} \frac{\tan^2 3x}{x \sin 2x} \div \frac{x^2}{x^2}$

$$\lim_{x \rightarrow 0} \frac{\left(\frac{\tan 3x}{x} \right)^2}{\frac{\sin 2x}{x}} = \frac{3^2}{2} = \frac{9}{2}$$

(5) $\lim_{x \rightarrow 0} \frac{3x^2 + \sin^2 x}{8x^2 - \tan^2 2x}$

Solution: $\lim_{x \rightarrow 0} \frac{3x^2 + \sin^2 x}{8x^2 - \tan^2 2x} \div \frac{x^2}{x^2}$

$$\lim_{x \rightarrow 0} \frac{3 + \left(\frac{\sin x}{x} \right)^2}{8 - \left(\frac{\tan 2x}{x} \right)^2} = \frac{3 + 1^2}{8 - 2^2} = 1$$

(6) $\lim_{x \rightarrow 0} \frac{\sin 4x \times \tan^2 5x}{x^2 \sin x}$

Solution: $\lim_{x \rightarrow 0} \frac{\sin 4x \times \tan^2 5x}{x^2 \sin x} \div \frac{x^3}{x^3}$

$$\lim_{x \rightarrow 0} \frac{\frac{\sin 4x}{x} \times \left(\frac{\tan 5x}{x} \right)^2}{\frac{\sin x}{x}} = \frac{4 \times 5^2}{1} = 100$$

IMPORTANT COLLORAIES:

① $\lim_{x \rightarrow 0} \cos x = 1$

② $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$

**Example (4)****Find the limit of the following functions:**

(1) $\lim_{x \rightarrow 0} \frac{\sin(1 - \cos x)}{1 - \cos x}$

Solution: $\lim_{x \rightarrow 0} \frac{\sin 4x \times \tan^2 5x}{x^2 \sin x}$

$$\lim_{x \rightarrow 0} \frac{\frac{\sin 4x}{x} \times \left(\frac{\tan 5x}{x}\right)^2}{\frac{\sin x}{x}} = \frac{4 \times 5^2}{1} = 100$$

(2) $\lim_{x \rightarrow 0} \frac{3 - 3 \cos^2 4x}{8x^2}$

Solution: $\lim_{x \rightarrow 0} \frac{3 - 3 \cos^2 4x}{8x^2} = \lim_{x \rightarrow 0} \frac{3(1 - \cos^2 4x)}{8x^2}$

$$\lim_{x \rightarrow 0} \frac{3 \sin^2 4x}{8x^2} = \frac{3}{8} \times \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{x}\right)^2 = \frac{3}{8} \times 4^2 = 6$$

(3) $\lim_{x \rightarrow 0} \frac{1 - (\sin x + \cos x)^2}{7x}$

Solution: $\lim_{x \rightarrow 0} \frac{1 - (\sin x + \cos x)^2}{7x} = \lim_{x \rightarrow 0} \frac{1 - (\sin^2 x + 2 \sin x \cos x + \cos^2 x)}{7x}$

$$\lim_{x \rightarrow 0} \frac{1 - (1 + 2 \sin x \cos x)}{7x}$$

$$\lim_{x \rightarrow 0} \frac{1 - 1 - 2 \sin x \cos x}{7x} = \lim_{x \rightarrow 0} \frac{-2 \sin x \cos x}{7x}$$

$$\frac{-2}{7} \times \lim_{x \rightarrow 0} \frac{\sin x \cos x}{x} = \frac{-2}{7} \times \lim_{x \rightarrow 0} \frac{\sin x}{x} \times \lim_{x \rightarrow 0} \cos x = \frac{-2}{7} \times 1 \times 1 = -\frac{2}{7}$$

(4) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\tan x}$

Solution: $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\tan x} = \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x} \times \frac{x}{\tan x}\right)$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} \times \lim_{x \rightarrow 0} \frac{x}{\tan x} = 0 \times 1 = 0$$

NOTE THAT:

❶ $\sin^2 x + \cos^2 x = 1$

❷ $1 - \sin^2 x = \cos^2 x$

❸ $1 - \cos^2 x = \sin^2 x$

**PRACTICE**

Q1: Evaluate $\lim_{x \rightarrow 0} \frac{\sin x}{\sin\left(\frac{x}{2}\right)}$.

- ☐ A 0
- ☐ B $\frac{1}{2}$
- ☐ C 1
- ☐ D 2

Q2: Find $\lim_{x \rightarrow -\frac{3}{2}} \frac{\sin(2x + 3)}{3(2x + 3)}$.

- ☐ A $\frac{1}{3}$
- ☐ B $-\frac{3}{2}$
- ☐ C $-\frac{1}{3}$
- ☐ D 0
- ☐ E $\frac{3}{2}$

Q3: Find $\lim_{x \rightarrow 0} \frac{\sin 5x \cos 4x}{8x}$.

- ☐ A 0
- ☐ B $\frac{1}{8}$
- ☐ C $\frac{1}{2}$
- ☐ D $\frac{5}{8}$

Q4: Find $\lim_{x \rightarrow 0} \frac{\sin^2 2x}{4x}$.



Q5: Find $\lim_{x \rightarrow 0} \frac{\tan^2 9x}{6x^2}$.

☐ A $\frac{1}{6}$

☐ B $\frac{27}{2}$

☐ C $\frac{3}{2}$

☐ D 0

Q6: Find $\lim_{x \rightarrow 0} \frac{4x^2}{\sin^2 5x}$.

☐ A $\frac{4}{25}$

☐ B $\frac{1}{5}$

☐ C 0

☐ D $\frac{4}{5}$

Q7: Find $\lim_{x \rightarrow 0} \frac{\tan^2 2x}{15x^2}$.

☐ A $\frac{4}{15}$

☐ B $-\frac{2}{15}$

☐ C $\frac{15}{2}$

☐ D $-\frac{15}{2}$

☐ E $\frac{15}{4}$



Q8: Determine $\lim_{x \rightarrow 0} \left(9 + \frac{1}{4x}\right) \sin 2x$.

- ☐ A 9
- ☐ B $\frac{19}{2}$
- ☐ C $\frac{1}{4}$
- ☐ D $\frac{1}{2}$

Q9: Find $\lim_{x \rightarrow 0} \frac{\cos^2 x + 6 \cos x - 7}{5 \sin^2 x}$.

- ☐ A $-\frac{4}{5}$
- ☐ B $-\frac{8}{5}$
- ☐ C $-\frac{6}{5}$
- ☐ D $-\frac{3}{5}$

Q10: Find $\lim_{x \rightarrow 0} \frac{\tan(\tan 7x)}{9x}$.

- ☐ A 9
- ☐ B $\frac{7}{9}$
- ☐ C 63
- ☐ D $\frac{9}{7}$
- ☐ E 7



Exercises 3 - 4



Complete:

$$\textcircled{1} \quad \lim_{x \rightarrow 0} \cos 3x = \dots\dots\dots$$

$$\textcircled{2} \quad \lim_{x \rightarrow \frac{\pi}{2}} \sin 2x = \dots\dots\dots$$

$$\textcircled{3} \quad \lim_{x \rightarrow 0} \tan x = \dots\dots\dots$$

$$\textcircled{4} \quad \lim_{x \rightarrow 0} \frac{\sin 7x}{x} = \dots\dots\dots$$

$$\textcircled{5} \quad \lim_{x \rightarrow 0} \tan \frac{3x}{4x} = \dots\dots\dots$$

$$\textcircled{6} \quad \lim_{x \rightarrow 5} \frac{\sin(x-5)}{3(x-5)} = \dots\dots\dots$$

$$\textcircled{7} \quad \lim_{x \rightarrow 0} \frac{\sin \pi x}{2x} = \dots\dots\dots$$

$$\textcircled{8} \quad \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} = \dots\dots\dots$$

$$\textcircled{9} \quad \lim_{x \rightarrow 0} \frac{3+2x}{\cos 4x} = \dots\dots\dots$$

$$\textcircled{10} \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{3x} = \dots\dots\dots$$

$$\textcircled{11} \quad \lim_{x \rightarrow 0} \frac{2x}{\tan 3x} = \dots\dots\dots$$

$$\textcircled{12} \quad \lim_{x \rightarrow 0} 3x \csc 2x = \dots\dots\dots$$

$$\textcircled{13} \quad \lim_{x \rightarrow 0} \frac{\sin^2 2x}{3x^2} = \dots\dots\dots$$

$$\textcircled{14} \quad \lim_{x \rightarrow 0} \frac{\sin^2 x \tan 3x}{4x^2} = \dots\dots\dots$$



Choose the correct answer from those given:

15 $\lim_{x \rightarrow 0} \frac{\sin 3x}{x} = \dots\dots\dots$

a 0

b $\frac{1}{3}$

c 1

d 3

16 $\lim_{x \rightarrow 0} \frac{\tan 4x}{5x} = \dots\dots\dots$

a 0

b $\frac{4}{5}$

c 1

d $\frac{5}{4}$

17 $\lim_{x \rightarrow 0} \frac{\sin 2x + 3 \tan x}{5x} = \dots\dots\dots$

a 1

b $\frac{5}{6}$

c $\frac{6}{5}$

d 2

18 $\lim_{x \rightarrow 0} \frac{\tan^2 2x}{x \sin 3x} = \dots\dots\dots$

a $\frac{4}{9}$

b $\frac{1}{2}$

c $\frac{2}{3}$

d $\frac{4}{3}$

19 $\lim_{x \rightarrow 0} \frac{\sin \frac{1}{2}x}{\sin \frac{3}{4}x} = \dots\dots\dots$

a $\frac{1}{6}$

b $\frac{3}{8}$

c $\frac{1}{2}$

d $\frac{2}{3}$

20 $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \dots\dots\dots$ where x in degree measure

a 1

b $\frac{\pi}{180}$

c $\frac{180}{\pi}$

d π



Find the following limits

$$21 \quad \lim_{x \rightarrow 0} \frac{\sin x}{5x}$$

$$23 \quad \lim_{x \rightarrow 0} \frac{\tan 2x}{x}$$

$$25 \quad \lim_{x \rightarrow 0} \frac{\sin x (1 - \cos x)}{x^2}$$

$$27 \quad \lim_{x \rightarrow 0} \frac{\sin^2 x}{x}$$

$$29 \quad \lim_{x \rightarrow 0} \frac{(1 - \cos^2 x)}{x^2}$$

$$31 \quad \lim_{x \rightarrow 0} \frac{x - x \cos x}{\sin^2 3x}$$

$$33 \quad \lim_{x \rightarrow \frac{1}{2}} \frac{x \cos(-2x+1)}{x^2 + x}$$

$$35 \quad \lim_{x \rightarrow 0} (1 + \cos x) \times \frac{1 - \cos x}{x^2}$$

$$37 \quad \lim_{x \rightarrow 0} \frac{\sin 5x^3 + \sin^3 5x}{2x^3}$$

$$39 \quad \lim_{x \rightarrow 0} \frac{2x^3 + x \sin 5x}{x^2 - \tan 3x^2}$$

$$41 \quad \lim_{x \rightarrow 0} \frac{\tan 2x + 5 \sin 3x}{2 \sin 3x - \tan 5x}$$

$$43 \quad \lim_{x \rightarrow 0} \frac{1 - \cos 3x}{\cos^2 2x - 1}$$

$$45 \quad \lim_{x \rightarrow 0} \frac{\tan 3x^2 + \sin^2 5x}{x^2}$$

$$47 \quad \lim_{x \rightarrow 0} \frac{\sin(\sin x)}{5 \sin x}$$

$$49 \quad \lim_{x \rightarrow 0} x (\csc 2x - \cot 3x)$$

$$22 \quad \lim_{x \rightarrow 0} \frac{\sin x}{\tan x}$$

$$24 \quad \lim_{x \rightarrow 0} \frac{3(1 - \cos x)}{x}$$

$$26 \quad \lim_{x \rightarrow 0} \frac{\cos x \tan x}{x}$$

$$28 \quad \lim_{x \rightarrow 0} \frac{\tan x}{x^2}$$

$$30 \quad \lim_{x \rightarrow 0} \frac{\cos x - 1}{\sin x}$$

$$32 \quad \lim_{x \rightarrow 0} \frac{1 - \tan x}{\sin x - \cos x}$$

$$34 \quad \lim_{x \rightarrow 0} \frac{x^2 - 3 \sin x}{x}$$

$$36 \quad \lim_{x \rightarrow 0} \frac{\tan 3x^2 + \sin^2 5x}{x^2}$$

$$38 \quad \lim_{x \rightarrow 0} \left(\frac{2x^2 + \sin 3x}{2x^2 + \tan 6x} \right)^4$$

$$40 \quad \lim_{x \rightarrow 0} \frac{1 - \cos x + \sin x}{1 - \cos x - \sin x}$$

$$42 \quad \lim_{x \rightarrow 0} \frac{x \tan 2x}{x^2 + \sin^2 3x}$$

$$44 \quad \lim_{x \rightarrow 0} \frac{2 - \cos 3x - \cos 4x}{x}$$

$$46 \quad \lim_{x \rightarrow 0} \frac{1 - \cos 3x}{\cos^2 5x - 1}$$

$$48 \quad \lim_{x \rightarrow 0} \frac{x}{\cos(\frac{\pi}{2} - x)}$$

**Lesson (2)****The Cosine Law****Lesson objectives****Related Links**

- Solve the cosine rule of any triangle.
- Use the cosine rule to solve the triangle.
- Solve Modulate and solving mathematical and life problems using the cosine rule.

Important Notes:

$$a^2 = b^2 + c^2 - 2bc \cos A \text{ In } \Delta ABC:$$

$$b^2 = c^2 + a^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Notes that

Use the cosine law if you have 3 sides or the ratio between the three sides.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \quad \cos B = \frac{c^2 + a^2 - b^2}{2ac}, \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

The cyclic quadrilateral

If $a : b : c = 2 : 3 : 4$, then:

Let $a = 2 \text{ m}$, $b = 3 \text{ m}$ & $c = 4 \text{ m}$

To prove that ABCD is a cyclic quadrilateral:

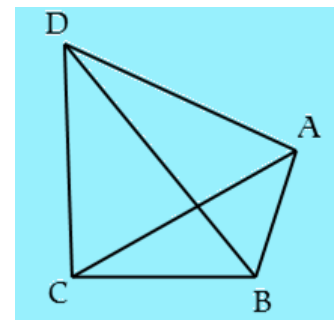
① The sum of measures of two opposite angles = 180°

$$m(\angle A) + m(\angle C) = 180^\circ$$

$$\cos A + \cos C = 0$$

$$m(\angle B) + m(\angle D) = 180^\circ$$

$$\cos B + \cos D = 0$$



② Two angles equal in measure drawn on one of its sides as a base

$$m(\angle BAC) = m(\angle BDC) \Rightarrow \cos(\angle BAC) = \cos(\angle BDC)$$

To find the measure of an angle, we prefer using the cosine law, because this law gives the kind of the angle, whether it is an acute or obtuse.

**Examples****Example (1)**

ABC is a triangle in which: $b = 30$ cm, $c = 14$ cm & $m(\angle A) = 60^\circ$ Find a

Solution

$$\therefore a^2 = b^2 + c^2 - 2bc \cos A$$

$$\therefore a^2 = 30^2 + 14^2 - 2 \times 30 \times 14 \times \cos 60^\circ = 676$$

$$\therefore a = \sqrt{676} = 26 \text{ cm.}$$

Example (2)

XYZ is a triangle in which: $x = 4$ cm, $y = 5$ cm, $z = 6$ cm, calculate the measure of its angles and its area.

Solution

$$\therefore \cos X = \frac{y^2 + z^2 - x^2}{2yz} = \frac{5^2 + 6^2 - 4^2}{2 \times 5 \times 6} = \frac{3}{4} \quad \therefore m(\angle X) = 41^\circ 24' 35''$$

$$\therefore \cos Y = \frac{x^2 + z^2 - y^2}{2xz} = \frac{4^2 + 6^2 - 5^2}{2 \times 4 \times 6} = \frac{9}{16} \quad \therefore m(\angle Y) = 55^\circ 46' 16''$$

$$\therefore m(\angle Z) = 180^\circ - [41^\circ 24' 35'' + 55^\circ 46' 16''] = 82^\circ 49' 9''$$

$$\therefore \text{Area of } \triangle XYZ = \frac{1}{2}xy \sin Z = \frac{1}{2} \times 4 \times 5 \times \sin 82^\circ 49' 9'' \simeq 10 \text{ cm}^2$$



Example (3)

In ΔABC : $\frac{1}{2} \sin A = \frac{1}{3} \sin B = \frac{1}{4} \sin C$. Calculate $m(\angle C)$

Solution

$$\therefore \frac{1}{2} \sin A = \frac{1}{3} \sin B = \frac{1}{4} \sin C$$

$$\therefore \frac{\sin A}{2} = \frac{\sin B}{3} = \frac{\sin C}{4}$$

$$\therefore \frac{2}{\sin A} = \frac{3}{\sin B} = \frac{4}{\sin C}$$

Let: $a = 2 \text{ m}$, $b = 3 \text{ m}$, $c = 4 \text{ m}$

$$\begin{aligned} \therefore \cos C &= \frac{a^2 + b^2 - c^2}{2ab} = \frac{(2 \text{ m})^2 + (3 \text{ m})^2 - (4 \text{ m})^2}{2 \times 2 \text{ m} \times 3 \text{ m}} \\ &= \frac{4 \text{ m}^2 + 9 \text{ m}^2 - 16 \text{ m}^2}{12 \text{ m}^2} = \frac{-3 \text{ m}^2}{12 \text{ m}^2} = \frac{-1}{4} \end{aligned}$$

$$\therefore m(\angle C) = 104^\circ 28' 39''$$

Example (4)

XYZ is a triangle in which: $x = 10.2 \text{ cm}$, $y = 14 \text{ cm}$ and $z = 16.8 \text{ cm}$.

Find $m(\angle Z)$ and the area of ΔXYZ to the nearest cm^2

Solution

$$\therefore \cos Z = \frac{x^2 + y^2 - z^2}{2xy} = \frac{10.2^2 + 14^2 - 16.8^2}{2 \times 10.2 \times 14} = \frac{89}{1428}$$

$$\therefore m(\angle Z) = 86^\circ 25' 36''$$

$$\therefore \text{Area of } \Delta XYZ = \frac{1}{2} x y \sin Z = \frac{1}{2} \times 10.2 \times 14 \times \sin 86^\circ 25' 36'' \simeq 71 \text{ cm}^2$$

**Example (5)**

In ΔABC : $a = 13$ cm , $b = 14$ cm , $c = 15$ cm.

Find the radius of its circumcircle.

Solution

$$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2 b c} = \frac{14^2 + 15^2 - 13^2}{2 \times 14 \times 15} = \frac{3}{5}$$

$$\therefore m(\angle A) = 53^\circ 7' 48''$$

$$\therefore \frac{a}{\sin A} = 2 r$$

$$\therefore 2 r = \frac{13}{\sin 53^\circ 7' 48''} = 16.25$$

$$\therefore r = 8.125 = 8\frac{1}{8} \text{ cm}$$

Example (6)

ABC is a triangle in which $a : b : c = 6 : 7 : 8$ Find $m(\angle B)$

Solution

Let: $a = 6$ m , $b = 7$ m , $c = 8$ m

$$\begin{aligned} \therefore \cos B &= \frac{a^2 + c^2 - b^2}{2 a c} = \frac{(6 \text{ m})^2 + (8 \text{ m})^2 - (7 \text{ m})^2}{2 \times 6 \text{ m} \times 8 \text{ m}} \\ &= \frac{36 \text{ m}^2 + 64 \text{ m}^2 - 49 \text{ m}^2}{96 \text{ m}^2} = \frac{51 \text{ m}^2}{96 \text{ m}^2} = \frac{17}{32} \end{aligned}$$

$$\therefore m(\angle B) = 57^\circ 54' 36''$$



Example (7)

ABC is a triangle in which D is the midpoint of $\overline{BC}$.

Let $m(\angle B) = 75^\circ$, $m(\angle A) = 60^\circ$ and $a = 8$ cm. Find the length of $\overline{AC}$ and $\overline{AD}$

Solution

$$\therefore m(\angle C) = 180^\circ - [75^\circ + 60^\circ] = 45^\circ$$

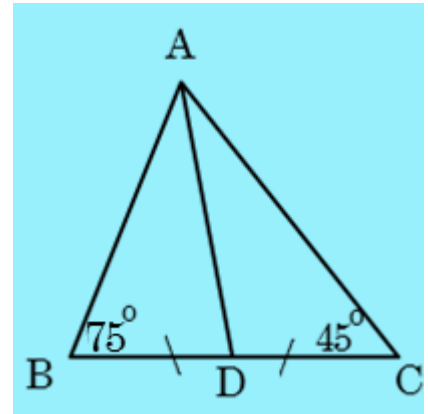
In $\triangle ABC$:

$$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\therefore \frac{BC}{\sin A} = \frac{AC}{\sin B} = \frac{AB}{\sin C}$$

$$\therefore \frac{8}{\sin 60^\circ} = \frac{AC}{\sin 75^\circ} = \frac{AB}{\sin 45^\circ}$$

$$\therefore AC = \frac{8 \times \sin 75^\circ}{\sin 60^\circ} \approx 9 \text{ cm}$$



In $\triangle ACD$:

$$\therefore m(\angle C) = 45^\circ, \quad CD = 4 \text{ cm} \quad \& \quad AC = 9 \text{ cm}$$

$$\therefore (AD)^2 = (AC)^2 + (CD)^2 - 2(AC)(CD)\cos C$$

$$\therefore (AD)^2 = 9^2 + 4^2 - 2 \times 9 \times 4 \times \cos 45^\circ = 46$$

$$\therefore AD = \sqrt{46} \approx 6.8 \text{ cm}$$

**Example (8)**

If: $(b + c + a)(b + c - a) = 3 b c$ in ΔABC , Prove that: $m(\angle A) = 60^\circ$

Solution

$$\because (b + c + a)(b + c - a) = 3 b c$$

$$\therefore (b + c)^2 - a^2 = 3 b c$$

$$\therefore b^2 + 2 b c + c^2 - a^2 = 3 b c$$

$$\therefore b^2 + c^2 - a^2 = 3 b c - 2 b c$$

$$\therefore b^2 + c^2 - a^2 = b c$$

$$\therefore \frac{b^2 + c^2 - a^2}{b c} = 1$$

$$\therefore \frac{b^2 + c^2 - a^2}{2 b c} = \frac{1}{2}$$

$$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2 b c}$$

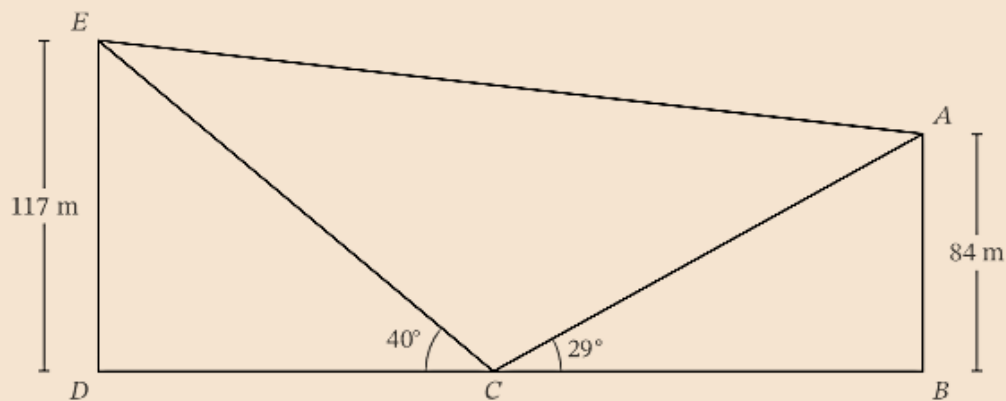
$$\therefore \cos A = \frac{1}{2}$$

$$\therefore m(\angle A) = 60^\circ$$

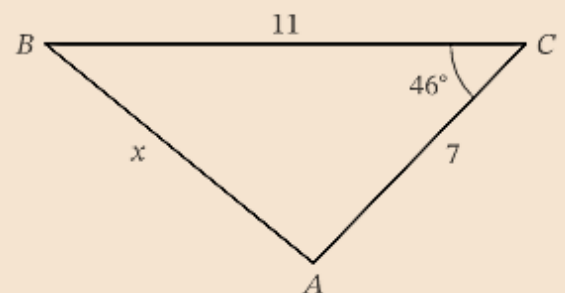


PRACTICE (1)

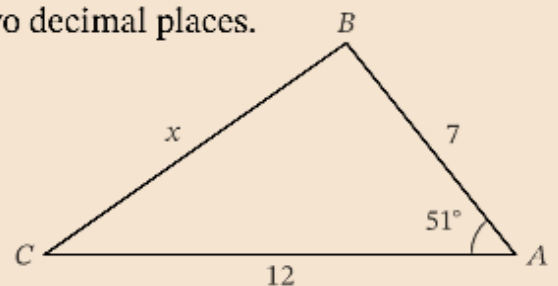
Q1: E and A are two hot air balloons flying at heights 117 m and 84 m respectively. The angles of depression at a point C on the ground from E and A are 40° and 29° respectively. Find the distance between the balloons giving the answer to the nearest meter.



Q2: In the given figure, find x . Give your answer to two decimal places.



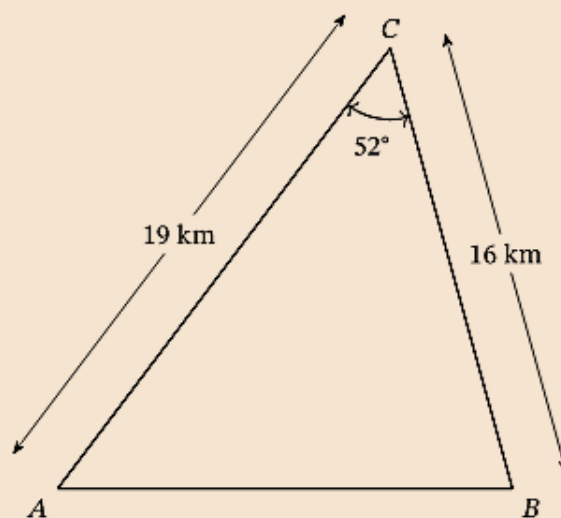
Q3: In the given figure, find x . Give your answer to two decimal places.





Q4: A , B , and C are three cities. Find the distance between cities A and B , giving your answer to the nearest kilometre.

- ☐ A 23 km
- ☐ B 12 km
- ☐ C 21 km
- ☐ D 16 km



Q5: ABC is a triangle where $BC = 38$ cm, $m\angle ACB = 60^\circ$ and the area is $399\sqrt{3}$ cm². Find the other lengths and angles giving lengths to one decimal place and angles to the nearest degree.

- ☐ A $AC = 42$ cm, $AB = 6.3$ cm, $m\angle BAC = 55^\circ 3'$, $m\angle ABC = 25^\circ 3'$
- ☐ B $AC = 42$ cm, $AB = 69.3$ cm, $m\angle BAC = 34^\circ 57'$, $m\angle ABC = 25^\circ 3'$
- ☐ C $AC = 42$ cm, $AB = 40.1$ cm, $m\angle BAC = 55^\circ 3'$, $m\angle ABC = 64^\circ 57'$
- ☐ D $AC = 10.5$ cm, $AB = 34$ cm, $m\angle BAC = 64^\circ 57'$, $m\angle ABC = 55^\circ 3'$

Q6: ABC is a triangle where $a = 18$ cm, $b = 10$ cm and $m\angle C = 76^\circ$. Find the measure of $\angle A$ giving the answer to the nearest second.

- ☐ A $63^\circ 37' 3''$
- ☐ B $66^\circ 21' 25''$
- ☐ C $72^\circ 5' 15''$
- ☐ D $31^\circ 54' 45''$



Q7: ABC is a triangle where $a = 67$ cm, $b = 49$ cm and the perimeter is 148 cm. Find the largest angle in ABC giving the answer to the nearest second.

- ☐ A $43^{\circ}28'9''$
- ☐ B $112^{\circ}21'48''$
- ☐ C $26^{\circ}41'51''$
- ☐ D $109^{\circ}50'$

Q8: ABC is a triangle where $a = 25$ cm, $b = 20$ cm and $c = 29$ cm. Find $m\angle A$ giving the answer to the nearest second.

- ☐ A $122^{\circ}4'31''$
- ☐ B $42^{\circ}40'42''$
- ☐ C $57^{\circ}55'29''$
- ☐ D $79^{\circ}23'50''$

Q9: ABC is a triangle where $\frac{1}{15} \sin A = \frac{1}{5} \sin B = \frac{1}{18} \sin C$. Find $m\angle C$ giving the answer to the nearest second.

- ☐ A $104^{\circ}16'49''$
- ☐ B $13^{\circ}58'56''$
- ☐ C $46^{\circ}27'28''$
- ☐ D $119^{\circ}33'36''$

Q10: ABC is a triangle where $P - a = 22$ cm, $P - b = 32$ cm, $P - c = 8$ cm and $2P = a + b + c$. Find the largest angle in the triangle giving the answer to the nearest second.

- ☐ A $99^{\circ}58'54''$
- ☐ B $46^{\circ}50'49''$
- ☐ C $94^{\circ}58'19''$
- ☐ D $33^{\circ}10'17''$

حمل الآن

مجاناً وحصرياً

المراجعة رقم (3)

اختبار شهر نوفمبر



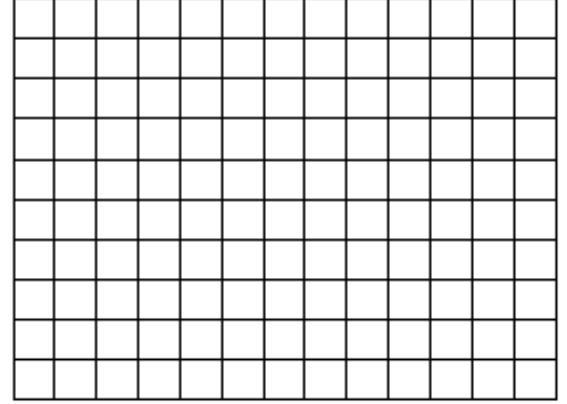


6 الرياضيات البحتة لغات - للصف الثاني الثانوي علمي الأداء الصفی الأسبوع السادس 6

Algebra: Exercises on Graphical representation of functions

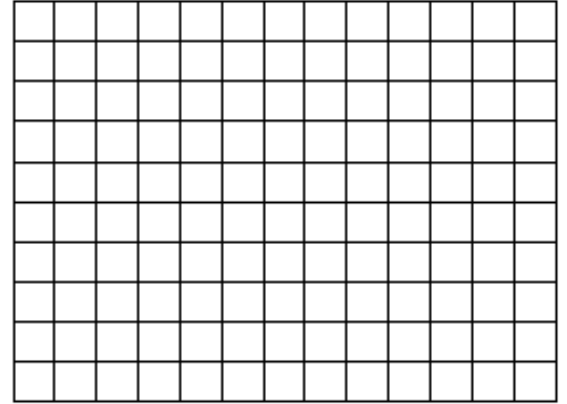
1) Graph the function $f : f(x) = \frac{1}{|x|} - 1$, from the graph deduce its domain, its range, discuss its monotony and mention its type (even, odd or otherwise).

Solu:
.....
.....
.....
.....
.....



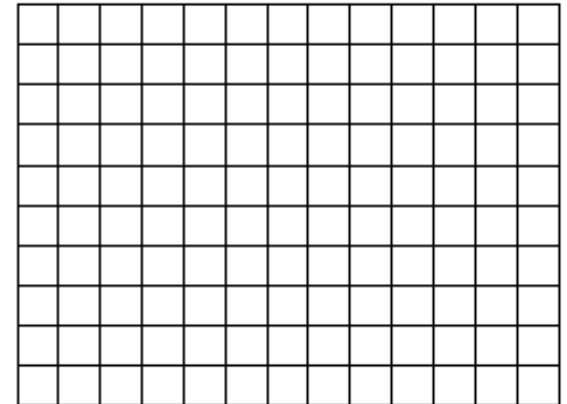
2) If the function $f(x) : f(x) = x^3$
Graph the function $g(x) : g(x) + f(x) = 1$,
Then deduce its domain, its range, discuss its monotony and mention its type (even, odd or otherwise).

Solu:
.....
.....
.....
.....
.....



3) Graph the function $f : f(x) = x^2 - 6x + 9$,
from the graph deduce its domain, its range,
discuss its monotony and mention its type
(even, odd or otherwise).

Solu:
.....
.....
.....
.....
.....





Algebra: Exercises on Solving Absolute Value Equations

4) Find in $\mathbb{R}$ the solution set of the following equation algebraically:

$$|x + 3| - x - 1 = 0$$

Solu:
.....
.....
.....

5) Find in $\mathbb{R}$ the solution set of the following equation algebraically:

$$|4x - 12| - |3 - x| = 15$$

Solu:
.....
.....
.....

6) Find in $\mathbb{R}$ the solution set of the following equation algebraically:

$$\sqrt{x^2 - 8x + 16} + |4 - x| = 6$$

Solu:
.....
.....
.....

Calculus: Finding the limit of a function algebraically and theorem (4)

7) Find: $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x^3 - 1}$

Solu:
.....
.....
.....

8) Find: $\lim_{x \rightarrow 2} \frac{2x^2 - x^3}{x^2 - 4}$

Solu:
.....
.....
.....



9) Find: $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 9} - 3}{x^2 + x}$

Solu:

.....

.....

10) Find: $\lim_{x \rightarrow 1} \frac{1 - x}{2 - \sqrt{x + 3}}$

Solu:

.....

.....

11) Find: $\lim_{x \rightarrow 1} \frac{x^3 - 3x + 2}{x^2 - 1}$

Solu:

.....

.....

12) Find: $\lim_{x \rightarrow -2} \frac{x^3 - 3x + 2}{x^3 + 2x^2 + x + 2}$

Solu:

.....

.....

13) Find: $\lim_{x \rightarrow 2} \frac{x^6 - x^5 - 32}{x^4 - 16}$

Solu:

.....

.....

14) Find: $\lim_{x \rightarrow -1} \frac{x^8 + x^3}{x^3 + 1}$

Solu:

.....

.....

15) Find: $\lim_{x \rightarrow -2} \frac{2x^6 - 128}{x^4 - 16}$

Solu:

.....

.....

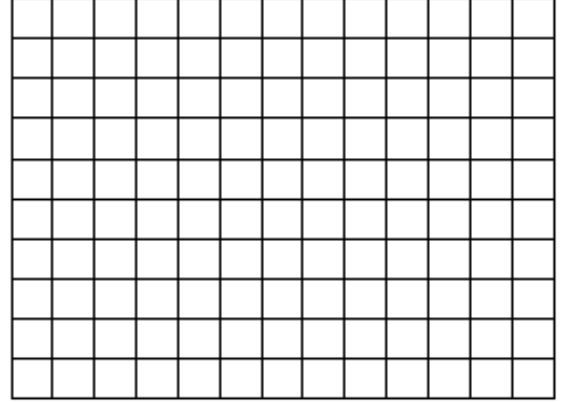


6 الرياضيات البحتة لغات - للصف الثاني الثانوي علمي الأداء المنزلي الأسبوع السادس 6

Algebra: Graphical representation of functions

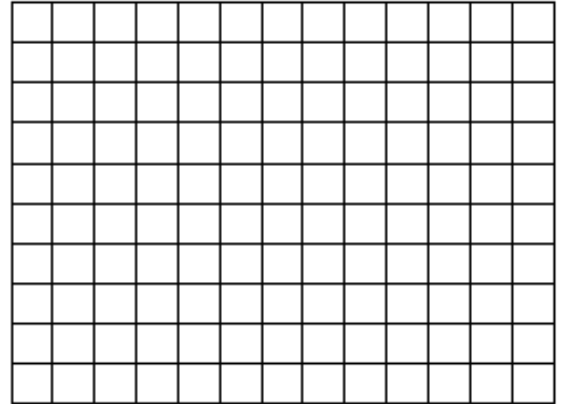
1) Graph the function $f: f(x) = (x - 2)^3 + 1$, from the graph deduce its domain, its range, discuss its monotony and mention if it is one to one or not.

Solu:
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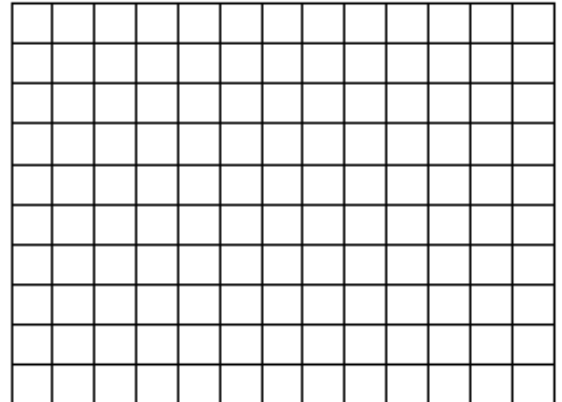
2) Use the graph of the function $f(x) = \frac{1}{x}$ to draw the graph the function $g(x): g(x) = \frac{-1}{x+2} + 1$, then deduce its domain, its range, discuss its monotony and mention its type (even, odd or otherwise).

Solu:
.....
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3) Graph the function $f: f(x) = x^3 - 1$, from the graph deduce its domain and its range.

Solu:
.....
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Algebra: Exercises on Solving Absolute Value Equations

4) Find in $\mathbb{R}$ the solution set of the following equation algebraically:

$$|x + 2| - x - 1 = 0$$

Solu:

.....

.....

.....

5) Find in $\mathbb{R}$ the solution set of the following equation algebraically:

$$|5x - 15| - |3 - x| = 20$$

Solu:

.....

.....

.....

6) Find in $\mathbb{R}$ the solution set of the following equation algebraically:

$$\sqrt{x^2 - 4x + 4} = 3$$

Solu:

.....

.....

.....

Calculus: Finding the limit of a function algebraically and theorem (4)

7) Find: $\lim_{x \rightarrow 3} \frac{3 + 2x - x^2}{27 - x^3}$

Solu:

.....

.....

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8) Find: $\lim_{x \rightarrow -1} \frac{x^2 - 1}{x^2 + x}$

Solu:

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9) Find: $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{\sqrt{5x - 6} - 3}$

Solu:

.....

.....

10) Find: $\lim_{x \rightarrow 0} \frac{\sqrt{x + 25} - 5}{x^2 + x}$

Solu:

.....

.....

11) Find: $\lim_{x \rightarrow 1} \frac{x^3 - 2x + 1}{x^2 + x - 2}$

Solu:

.....

.....

12) Find: $\lim_{x \rightarrow -3} \frac{x^3 - 6x + 9}{x^3 + 3x^2 + x + 3}$

Solu:

.....

.....

13) Find: $\lim_{x \rightarrow 1} \frac{x^7 + x^5 - 2}{x^6 + 1}$

Solu:

.....

.....

14) Find: $\lim_{x \rightarrow -1} \frac{x^{10} - x^4}{x^3 + 1}$

Solu:

.....

.....

15) Find: $\lim_{x \rightarrow -2} \frac{8x^6 - 512}{x^4 - 16}$

Solu:

.....

.....



7 الرياضيات البحتة لغات - للصف الثاني الثانوي علمي الأداء الصفی الأسبوع السابع 7

Algebra: Exercises on Solving Absolute Value Equations

1) Find in R the solution set of the following equation:

$$|4x - 2| - 3x + 1 = 0$$

Sol.:

.....

.....

.....

2) Find in R the solution set of the following equation:

$$x|x + 1| = 12$$

Sol.:

.....

.....

.....

3) Find in R the solution set of the following inequality:

$$|x + 5| < 4$$

Sol.:

.....

.....

.....

4) Find in R the solution set of the following inequality:

$$|2x + 4| > 8$$

Sol.:

.....

.....

.....

5) Find in R the solution set of the following inequality:

$$\sqrt{x^2 - 12x + 36} \leq 5$$

Sol.:

.....

.....

.....



Calculus: Exercises on theorem (4)

6) Find: $\lim_{x \rightarrow 1} \frac{\sqrt[7]{x} - 1}{\sqrt[5]{x} - 1}$

Sol.:

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7) Find: $\lim_{h \rightarrow 0} \frac{(x + h)^8 - x^8}{h}$

Sol.:

.....

.....

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8) Find: $\lim_{x \rightarrow -1} \frac{x^8 - 1}{x^3 + x^2}$

Sol.:

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.....

.....

9) Find: $\lim_{x \rightarrow 1} \frac{x^2 - 7x + 6}{x^9 - 1}$

Sol.:

.....

.....

.....

10) Find: $\lim_{x \rightarrow \frac{1}{3}} \frac{243x^4 - 3}{3x - 1}$

Sol.:

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Calculus: Limit of a function at infinity

11) Find: $\lim_{x \rightarrow \infty} (7x^2 + 4x + 1)$

Sol.:

.....

.....

.....

12) Find: $\lim_{x \rightarrow \infty} \frac{3x^{-3} + 8x^{-2} - 10}{2x^{-3} + 4x^{-2} + 5}$

Sol.:

.....

.....

.....

13) Find: $\lim_{x \rightarrow \infty} \frac{6x^4 + 4x^2 - 3}{5x^5 + x - 1}$

Sol.:

.....

.....

.....

14) Find: $\lim_{x \rightarrow \infty} \frac{8x^4 + 7x - 6}{4x^4 + 3}$

Sol.:

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.....

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15) Find: $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 + 1}}{4x - 3}$

Sol.:

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7 الرياضيات البحتة لغات - للصف الثاني الثانوي علمي الأداء المنزلي الأسبوع السابع 7

Algebra: Exercises on Solving Absolute Value Equations

1) Find in R the solution set of the following equation:

$$|3x - 2| - 2x - 1 = 0$$

Sol.:

.....

.....

.....

2) Find in R the solution set of the following equation:

$$x|x - 1| = 6$$

Sol.:

.....

.....

.....

3) Find in R the solution set of the following inequality:

$$|x - 2| < 3$$

Sol.:

.....

.....

.....

4) Find in R the solution set of the following inequality:

$$|2x + 5| > 3$$

Sol.:

.....

.....

.....

5) Find in R the solution set of the following inequality:

$$\sqrt{x^2 + 10x + 25} \leq 9$$

Sol.:

.....

.....

.....



Calculus: Exercises on theorem (4)

6) Find: $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{\sqrt[3]{x} - 1}$

Sol.:

.....

.....

.....

7) Find: $\lim_{h \rightarrow 0} \frac{(x+h)^7 - x^7}{h}$

Sol.:

.....

.....

.....

8) Find: $\lim_{x \rightarrow -1} \frac{x^6 - 1}{x^2 + x}$

Sol.:

.....

.....

9) Find: $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x^7 - 1}$

Sol.:

.....

.....

.....

10) Find: $\lim_{x \rightarrow \frac{1}{2}} \frac{128x^5 - 4}{2x - 1}$

Sol.:

.....

.....

.....



Calculus: Limit of a function at infinity

11) Find: $\lim_{x \rightarrow \infty} (3x^2 + 5x + 2)$

Sol.:

.....

.....

.....

12) Find: $\lim_{x \rightarrow \infty} \frac{7x^{-3} + 9x^{-2} - 12}{8x^{-3} + 2x^{-2} + 3}$

Sol.:

.....

.....

.....

13) Find: $\lim_{x \rightarrow \infty} \frac{8x^4 + 2x^2 - 7}{2x^5 + x - 2}$

Sol.:

.....

.....

.....

14) Find: $\lim_{x \rightarrow \infty} \frac{5x^4 + 2x - 1}{6x^4 + 13}$

Sol.:

.....

.....

.....

15) Find: $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^2 + 17}}{3x - 5}$

Sol.:

.....

.....

.....



7 الرياضيات البحتة لغات - للصف الثاني الثانوي علمي التقييمات الأسبوعية الأسبوع السابع 7

The first group

1) Find in R the solution set of the following equation:

$$|5x - 2| - 2x - 1 = 0$$

Solu:

.....

.....

.....

2) Find in R the solution set of the following inequality:

$$|3x + 6| > 9$$

Solu:

.....

.....

.....

3) Find in R the solution set of the following inequality:

$$|x + 7| < 5$$

Solu:

.....

.....

.....

4) Find: $\lim_{x \rightarrow -1} \frac{x^7 + 1}{x^4 + x^3}$

Solu:

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.....

5) Find: $\lim_{x \rightarrow \infty} \frac{12x^5 - 8x - 7}{3x^5 + 8}$

Solu:

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The second group

1) Find in $\mathbb{R}$ the solution set of the following equation:

$$|4x - 3| - 3x + 1 = 0$$

Solu:

.....

.....

.....

2) Find in $\mathbb{R}$ the solution set of the following inequality:

$$|5x - 10| > 5$$

Solu:

.....

.....

.....

3) Find in $\mathbb{R}$ the solution set of the following inequality:

$$|x + 6| \leq 7$$

Solu:

.....

.....

.....

4) Find: $\lim_{x \rightarrow -1} \frac{x^9 + 1}{x^5 + x^2}$

Solu:

.....

.....

.....

5) Find: $\lim_{x \rightarrow \infty} \frac{4x^5 - 7x - 1}{8x^5 + 5}$

Solu:

.....

.....

.....



The third group

1) Find in $\mathbb{R}$ the solution set of the following equation:

$$|2x - 1| - 3x + 4 = 0$$

Solu:

.....

.....

.....

2) Find in $\mathbb{R}$ the solution set of the following inequality:

$$|4x + 12| > 8$$

Solu:

.....

.....

.....

3) Find in $\mathbb{R}$ the solution set of the following inequality:

$$|x + 1| \leq 3$$

Solu:

.....

.....

.....

4) Find: $\lim_{x \rightarrow -1} \frac{x^5 + 1}{x^3 + x^2}$

Solu:

.....

.....

.....

5) Find: $\lim_{x \rightarrow \infty} \frac{x^5 - 4x - 9}{4x^5 + 2}$

Solu:

.....

.....

.....



8 الرياضيات البحتة لغات - للصف الثاني الثانوي علمي الأداء الصفی الأسبوع الثامن 8

Algebra: Exercises on Unit one

1) If the function $f : f(x) = \sqrt{x+1}$, the function $g : g(x) = x+1$, then find $(f \circ g)(2)$

Solu:

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2) Prove that the function $f : f(x) = (x+5)^3 - 4$ is one to one function

Solu:

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3) Find graphically in R the solution set of the following equation, then satisfy the solution algebraically $|2x+1| = 1-x$

Solu:

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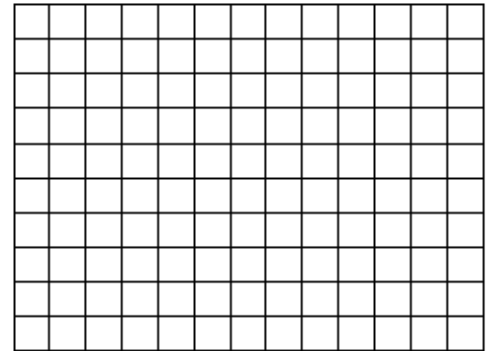
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4) Determine the domain of the function $f : f(x) = \frac{x^2 + 4}{\sqrt{2x - 6}}$

Solu:

5) Find in $\mathbb{R}$ the values of x which satisfy positive values for the expression $(|x - 4| - 7)$

Solu:

6) If the function $f : f(x) = x \tan x - \sin^2 x$, then mention if the function is even, odd or otherwise (give reason)

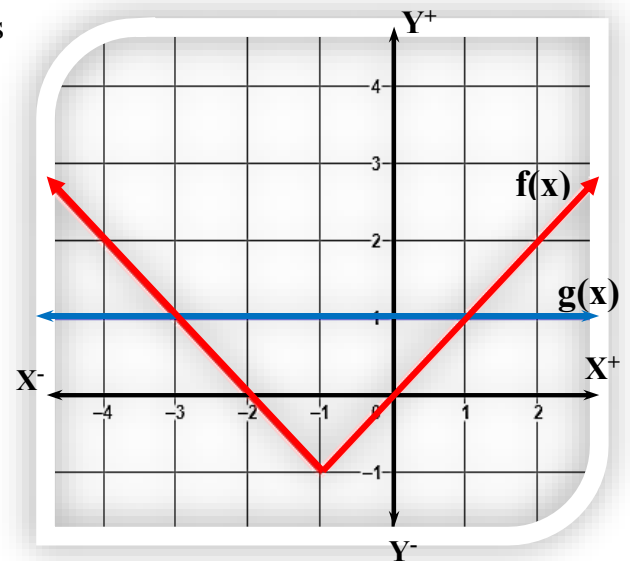
Solu:

7) The opposite figure shows the curves of the functions

$f(x)$, $g(x)$. Find in $\mathbb{R}$ the solution set of the inequality

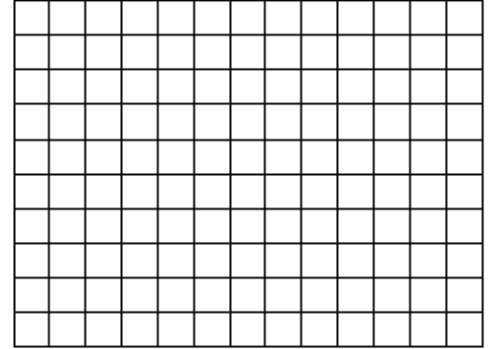
$f(x) \leq g(x)$

Solu:





8) Graph the function $f : f(x) = 2 - \frac{1}{x-1}$, from the graph deduce its domain, its range, discuss its monotony and mention if it is one to one or not.



Solu:
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Algebra: Exercises on Rational exponents.

9) Find in $\mathbb{R}$ the solution set of the following equation: $(x + 25)^{\frac{4}{3}} = 81$

Solu:
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.....
.....

10) Find in $\mathbb{R}$ the solution set of the following equation: $(2x + 3)^{\frac{3}{4}} - 27 = 0$

Solu:
.....
.....
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.....
.....



Calculus: Exercises on Limit of a function at infinity

11) Find: $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 3x}{x + 5} - \frac{x^2 + 7}{x + 5} \right)$

Solu:

.....

.....

.....

12) Find: $\lim_{x \rightarrow \infty} (\sqrt{4x^2 - 2x} - 2x)$

Solu:

.....

.....

.....

Exercises on Cosine Rule

13) ABC is a triangle in which: $a = 6$ cm, $c = 10$ cm, $m(\angle B) = 120^\circ$, find the length of b .

Solu:

.....

.....

.....

14) ABC is a triangle in which: $a = 3$ cm, $m(\angle B) = 120^\circ$ and its surface area = $10\sqrt{3}$ cm²,
find the length of each of c , b .

Solu:

.....

.....

.....

15) ABC is a triangle in which: $\cos A = \frac{2}{5}$, $b = 2.5$ cm and $c = 2$ cm.

Prove that ΔABC is an isosceles triangle

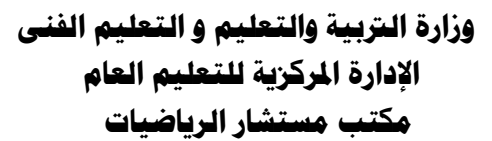
Solu:

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Algebra: Exercises on Unit one

Solu:

Solu:

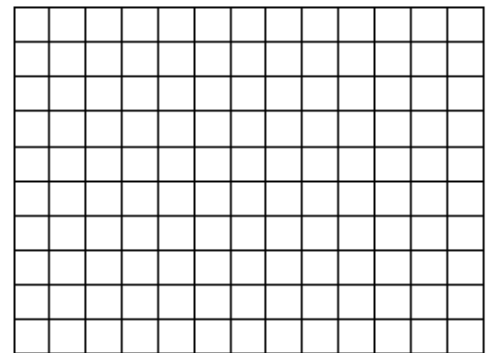
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[illegible]



4) Determine the domain of the function $f : f(x) = \frac{x^3 - x + 1}{\sqrt{2x - 2}}$

Solu:

.....

.....

.....

.....

.....

5) Find in $\mathbb{R}$ the values of x which satisfy negative values for the expression $(|x - 4| - 7)$

Solu:

.....

.....

.....

.....

.....

6) If the function $f : f(x) = x \sin x - \cos x$, then mention if the function is even, odd or otherwise (give reason)

Solu:

.....

.....

.....

.....

.....

7) The opposite figure shows the curves of the functions $f(x)$, $g(x)$. Find in $\mathbb{R}$ the solution set of the inequality $f(x) \geq g(x)$

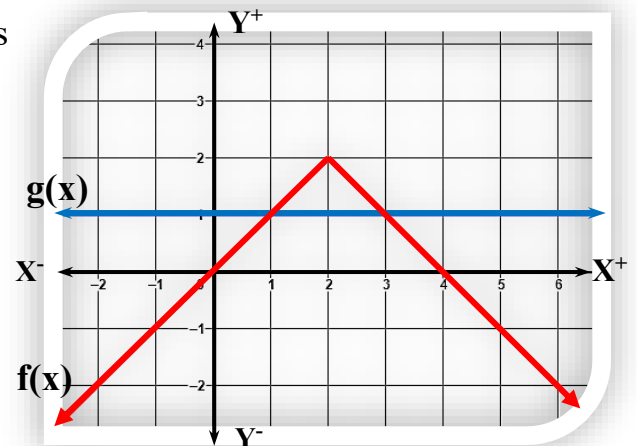
Solu:

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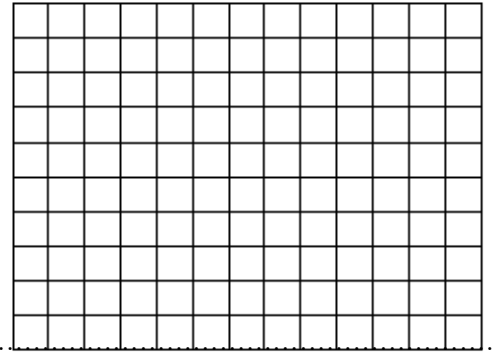
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8) Graph the function $f : f(x) = 1 - (x + 1)^3$, from the graph deduce its domain, its range, discuss its monotony and mention if it is one to one or not.



Solu:

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.....

.....

.....

Algebra: Exercises on Rational exponents.

9) Find in R the solution set of the following equation: $(x - 1)^{\frac{4}{3}} = 16$

Solu:

.....

.....

.....

.....

10) Find in R the solution set of the following equation: $(2x + 3)^{\frac{3}{4}} - 8 = 0$

Solu:

.....

.....

.....

.....

.....

Calculus: Exercises on Limit of a function at infinity

11) Find: $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 4x}{x + 2} - \frac{x^2 + 5}{x + 2} \right)$

Solu:

.....

.....

.....



12) Find: $\lim_{x \rightarrow \infty} (\sqrt{9x^2 + 6x} - 3x)$

Solu:

.....

.....

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.....

.....

Exercises on Cosine Rule

13) ABC is a triangle in which: $a = 9$ cm, $c = 15$ cm, $m(\angle B) = 120^\circ$, find the length of b.

Solu:

.....

.....

.....

.....

.....

14) ABC is a triangle in which: $b = 9$ cm, $m(\angle B) = 30^\circ$ and its surface area = 27 cm^2 ,
find the length of each of c , a.

Solu:

.....

.....

.....

.....

.....

15) ABC is a triangle in which: $\cos A = \frac{2}{5}$, $b = 5$ cm and $c = 4$ cm.

Prove that ΔABC is an isosceles triangle

Solu:

.....

.....

.....

.....

.....



8 الرياضيات البحتة لغات - للصف الثاني الثانوي علمي التقييمات الأسبوعية الأسبوع الثامن 8

The First Group

1) Prove that the function $f : f(x) = \frac{3x}{x+2}$ is one to one function

Solu:

2) Find graphically in R the solution set of the following equation , then satisfy the solution algebraically $|x + 4| = 1 - 2x$

Solu:

3) Find in R the solution set of the following equation: $(x + 2)^{\frac{3}{4}} - 1 = 0$

Solu:

4) Find: $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x}{x + 3} - \frac{x^2 + 1}{x + 3} \right)$

Solu:

5) ABC is a triangle in which: $a = 12$ cm, $c = 20$ cm, $m(\angle B) = 120^\circ$ find the length of b.

Solu:



The Second Group

1) Prove that the function $f : f(x) = \frac{5x}{x-2}$ is one to one function

Solu:
.....
.....
.....

2) Find graphically in R the solution set of the following equation , then satisfy the solution algebraically $|x - 3| = 3 - 2x$

Solu:
.....
.....
.....

3) Find in R the solution set of the following equation: $(x + 15)^{\frac{3}{4}} - 8 = 0$

Solu:
.....
.....
.....

4) Find: $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 4}{x + 6} - \frac{x^2 - 3x}{x + 6} \right)$

Solu:
.....
.....
.....
.....

5) ABC is a triangle in which: $a = 15$ cm, $c = 25$ cm, $m(\angle B) = 120^\circ$ find the length of b.

Solu:
.....
.....
.....



The Third Group

1) Prove that the function $f : f(x) = \frac{4x}{x+3}$ is one to one function

Solu:
.....
.....
.....

2) Find graphically in R the solution set of the following equation, then satisfy the solution algebraically $|x - 6| = 3 - 2x$

Solu:
.....
.....
.....

3) Find in R the solution set of the following equation: $(x + 8)^{\frac{3}{4}} - 27 = 0$

Solu:
.....
.....
.....

4) Find: $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 9}{x + 4} - \frac{x^2 - 5x}{x + 4} \right)$

Solu:
.....
.....
.....

5) ABC is a triangle in which: $a = 18$ cm, $c = 30$ cm, $m(\angle B) = 120^\circ$ find the length of b.

Solu:
.....
.....
.....



9 الرياضيات البحتة لغات - للصف الثاني الثانوي علمي الأداء الصفی الأسبوع التاسع 9

Exercises on exponential function and its applications

1) Find in the simplest form each of:

a) $\sqrt[3]{-64 a^9 b^{12}}$

b) $\sqrt[4]{81 x^{12} y^{16}}$

c) $\sqrt{\frac{1}{16} x^6 y^{10}}$

Solu:

.....

.....

.....

.....

2) Determine which of the following is an exponential function, then write its exponent and its base:

a) $f(x) = \frac{6}{5} (2)^x$

b) $f(x) = (-3)^x$

c) $f(x) = \left(\frac{1}{5}\right)^x - 1$

Solu:

.....

.....

.....

.....

3) If the function $f: f(x) = 3^x$

Find each of the following : $f(-1)$, $f(2)$, $f(x-1)$, $f(x) \times f(-x)$

Solu:

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.....

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.....

4) If the point A (2, 9) belongs to the curve of the function $f: f(x) = \left(\frac{1}{2}\right)^{3x-8} + k$

then find the value of k

Solu:

.....

.....

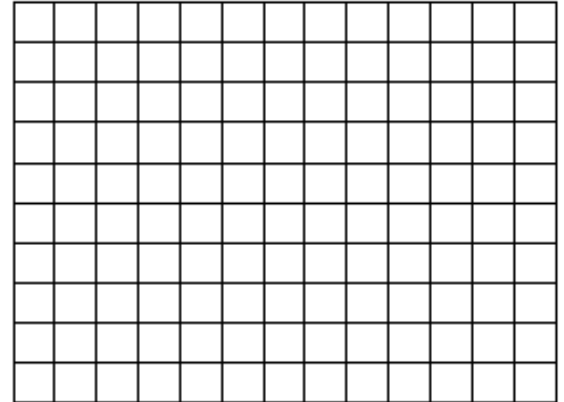
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5) Graph the function $f : f(x) = (3)^x$, from the graph deduce its domain, its range, discuss if it is increasing or decreasing and mention if it is one to one or not.

Solu:

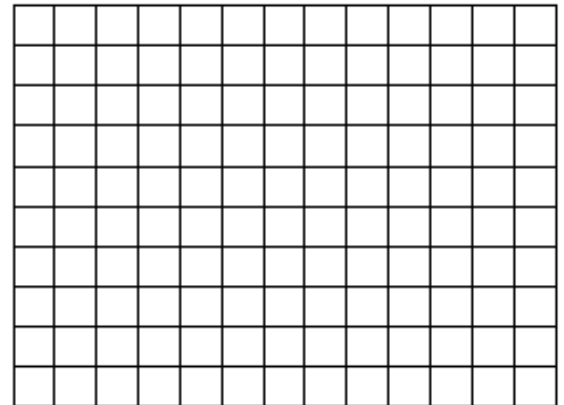
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6) Graph the function $f : f(x) = -\left(\frac{1}{2}\right)^x$, from the graph deduce its domain, its range, discuss if it is increasing or decreasing and mention if it is one to one or not.

Solu:

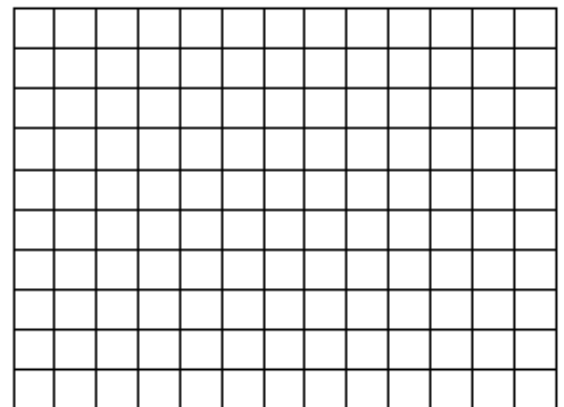
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7) Graph the function $f : f(x) = -(2)^x + 2$, from the graph deduce its domain, its range, discuss if it is increasing or decreasing and mention if it is one to one or not.

Solu:

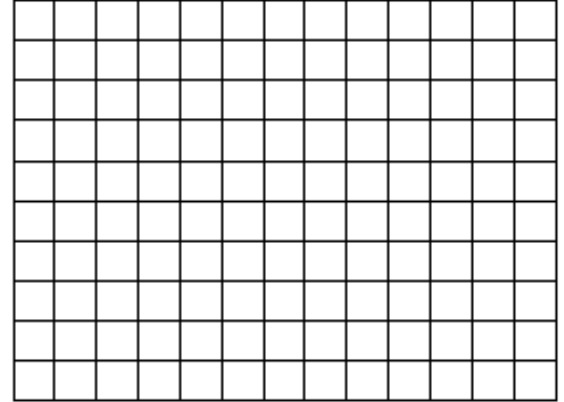
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8) Graph the function $f: f(x) = (3)^{x+1}$, from the graph deduce its domain, its range, discuss if it is increasing or decreasing and mention if it is one to one or not.

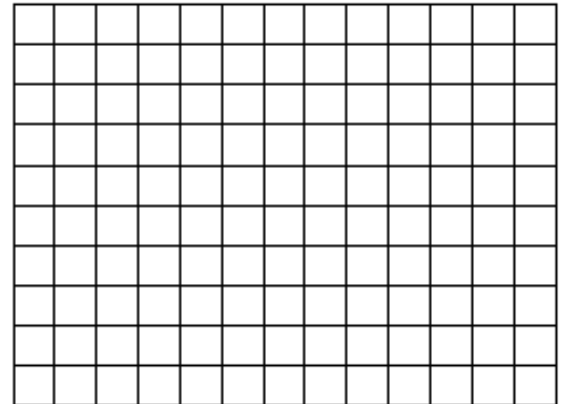
Solu:
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9) Graph the function $f: f(x) = \left(\frac{1}{2}\right)^{x-2} - 1$, from the graph deduce its domain, its range, discuss if it is increasing or decreasing and mention if it is one to one or not.

Solu:
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Calculus: Exercises on Limits of trigonometric functions

10) Find: $\lim_{x \rightarrow 0} \frac{4}{x} \sin \frac{x}{2}$

Solu:

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11) Find: $\lim_{x \rightarrow 0} \frac{\sin x (1 - \cos x)}{x^2}$

Solu:

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12) Find: $\lim_{x \rightarrow 0} \frac{\tan 3x^2 + \sin^2 2x}{x^2}$

Solu:

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Exercises on Finding the measure of an angle of a triangle given the lengths of its sides

13) ABC is a triangle in which : $a = 7 \text{ cm}$, $b = 6 \text{ cm}$, $c = 9 \text{ cm}$, find $m(\angle B)$.

Solu:

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14) In ΔABC : $\frac{1}{3} \sin A = \frac{1}{5} \sin B = \frac{1}{7} \sin C$, find $m(\angle C)$.

Solu:

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15) ABC is a triangle in which : $a = 11 \text{ cm}$, $b = 6 \text{ cm}$, $c = 10 \text{ cm}$, find the circumference of the circumcircle of the triangle ABC to the nearest centimeter.

Solu:

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9 الرياضيات البحتة لغات - للصف الثاني الثانوي علمي الأداء المنزلي الأسبوع التاسع 9

Exercises on exponential function and its applications

1) Find in the simplest form each of:

a) $\sqrt[3]{-27 a^3 b^6}$

b) $\sqrt[4]{16 x^4 y^8}$

c) $\sqrt[5]{32 x^5 y^{15}}$

Solu:

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2) Determine which of the following is an exponential function, then write its exponent and its base:

a) $f(x) = \frac{8}{9} (3)^x$

b) $f(x) = (-4)^x$

c) $f(x) = \left(\frac{1}{2}\right)^{x-1}$

Solu:

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3) If the function $f : f(x) = 2^x$

Find each of the following : $f(-2)$, $f(3)$, $f(x+1)$, $f(x) \times f(-x)$

Solu:

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4) If the point A (1 , 7) belongs to the curve of the function $f : f(x) = \left(\frac{1}{3}\right)^{2x-3} + k$
then find the value of k

Solu:

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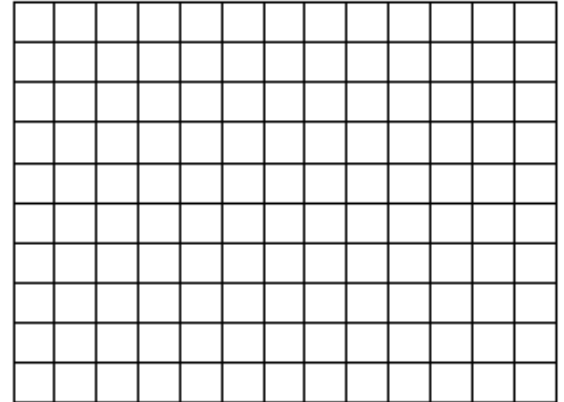
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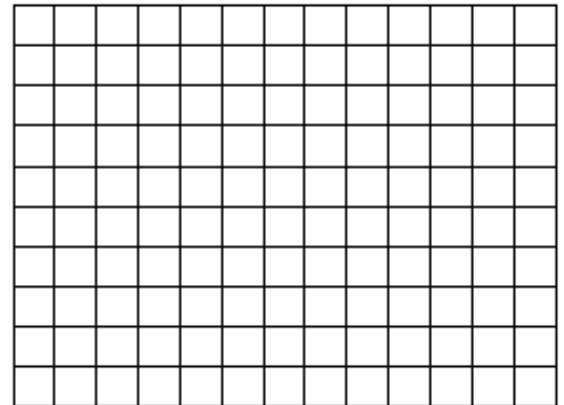
5) Graph the function $f : f(x) = (2)^x$, from the graph deduce its domain, its range, discuss if it is increasing or decreasing and mention if it is one to one or not.

Solu:
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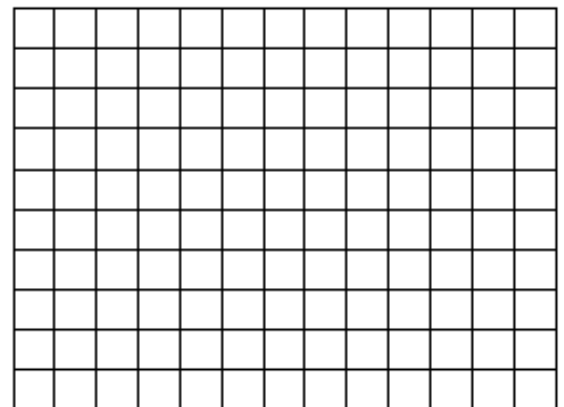
6) Graph the function $f : f(x) = -\left(\frac{1}{3}\right)^x$, from the graph deduce its domain, its range, discuss if it is increasing or decreasing and mention if it is one to one or not.

Solu:
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7) Graph the function $f : f(x) = -(3)^x + 1$, from the graph deduce its domain, its range, discuss if it is increasing or decreasing and mention if it is one to one or not.

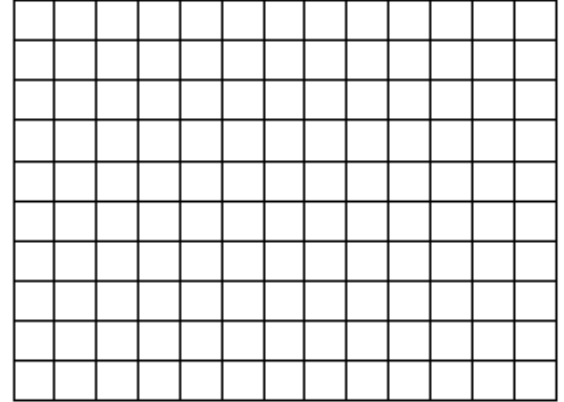
Solu:
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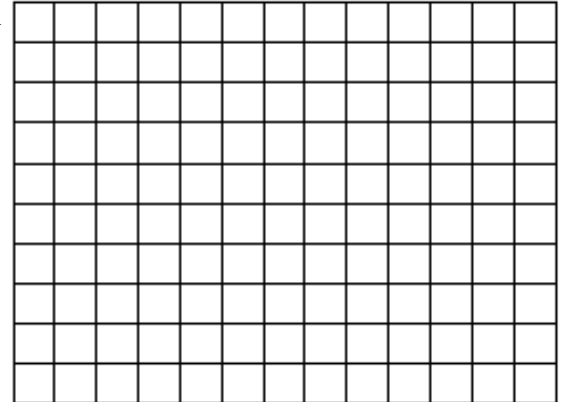
8) Graph the function $f : f(x) = (2)^{x-1}$, from the graph deduce its domain, its range, discuss if it is increasing or decreasing and mention if it is one to one or not.

Solu:
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9) Graph the function $f : f(x) = \left(\frac{1}{2}\right)^{x+2} + 1$, from the graph deduce its domain, its range, discuss if it is increasing or decreasing and mention if it is one to one or not.

Solu:
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Calculus: Exercises on Limits of trigonometric functions

10) Find: $\lim_{x \rightarrow 0} \frac{6}{x} \sin \frac{x}{3}$

Solu:

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11) Find: $\lim_{x \rightarrow 0} (1 - \cos x) \cot x$

Solu:

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12) Find: $\lim_{x \rightarrow 0} \frac{\sin 5x^2 + \tan^2 3x}{x^2}$

Solu:

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Exercises on Finding the measure of an angle of a triangle given the lengths of its sides

13) ABC is a triangle in which : $a = 8 \text{ cm}$, $b = 9 \text{ cm}$, $c = 10 \text{ cm}$, find $m(\angle B)$.

Solu:

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14) In ΔABC : $\frac{1}{4} \sin A = \frac{1}{5} \sin B = \frac{1}{7} \sin C$, find $m(\angle C)$.

Solu:

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15) ABC is a triangle in which : $a = 12 \text{ cm}$, $b = 7 \text{ cm}$, $c = 8 \text{ cm}$, find the circumference of the circumcircle of the triangle ABC to the nearest centimeter.

Solu:

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٩ الرياضيات البحتة لغات - للصف الثاني الثانوي علمي التقييمات الأسبوعية الأسبوع التاسع ٩

The First Group

1) If the function $f : f(x) = \left(\frac{1}{3}\right)^x$

Find each of the following : $f(-2)$, $f(3)$, $f(x+2)$, $f(x) \times f(-x)$

Solu:

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2) If the point A (4 , 30) belongs to the curve of the function $f : f(x) = \left(\frac{1}{5}\right)^{x-6} + k$

then find the value of k

Solu:

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3) Graph the function $f : f(x) = -(2)^x$, from the graph deduce its domain, its range, discuss if it is increasing or decreasing and mention if it is one to one or not.

Solu:

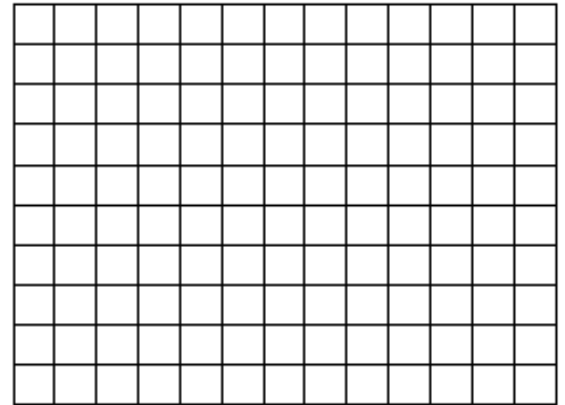
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4) Find: $\lim_{x \rightarrow 0} \frac{\tan 2x^2 + \sin^2 x}{x^2}$

Solu:

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5) In ΔABC if : $a = 5$ cm , $b = 6$ cm , $c = 8$ cm , , find $m(\angle B)$.

Solu:

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The Second Group

1) If the function $f : f(x) = \left(\frac{1}{5}\right)^x$

Find each of the following : $f(-1)$, $f(2)$, $f(x-2)$, $f(x) \times f(-x)$

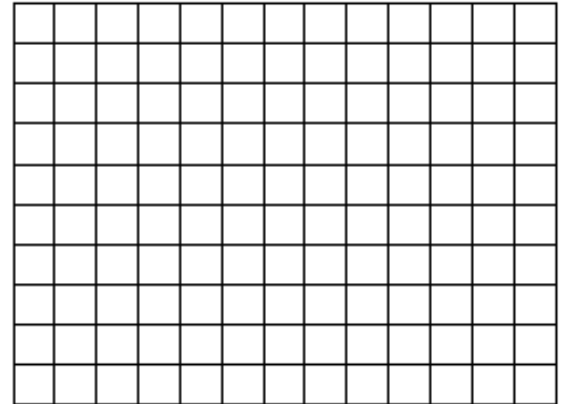
Solu:
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2) If the point A $(-1, 15)$ belongs to the curve of the function $f : f(x) = (3)^{4x+6} + k$
then find the value of k

Solu:
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3) Graph the function $f : f(x) = -\left(\frac{1}{2}\right)^x$, from the graph
deduce its domain, its range, discuss if it is increasing
or decreasing and mention if it is one to one or not.

Solu:
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4) Find: $\lim_{x \rightarrow 0} \frac{\tan 2x^2 - 3 \sin^2 x}{x^2}$

Solu:
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5) In ΔABC if : $a = 10 \text{ cm}$, $b = 11 \text{ cm}$, $c = 13 \text{ cm}$, , find $m(\angle B)$.

Solu:
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The Third Group

1) If the function $f : f(x) = \left(\frac{1}{4}\right)^x$

Find each of the following : $f(-1)$, $f(3)$, $f(x-1)$, $f(x) \times f(-x)$

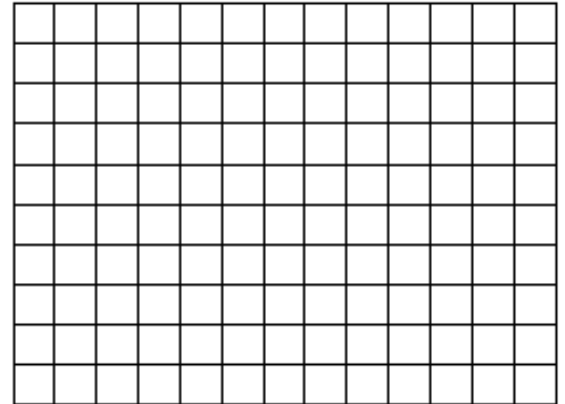
Solu:
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2) If the point A $(-1, 10)$ belongs to the curve of the function $f : f(x) = (2)^{3x+5} + k$
then find the value of k

Solu:
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3) Graph the function $f : f(x) = -(3)^x$, from the graph deduce its domain, its range, discuss if it is increasing or decreasing and mention if it is one to one or not.

Solu:
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4) Find: $\lim_{x \rightarrow 0} \frac{\tan 7x^2 + 2 \sin^2 x}{x^2}$

Solu:
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5) In ΔABC if : $a = 3 \text{ cm}$, $b = 6 \text{ cm}$, $c = 8 \text{ cm}$, , find $m(\angle B)$.

Solu:
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كيفية طباعة صفحات معينة من ملف معين

مثلا ازاي نطبع الصفحات من صفحة 4 الى صفحة 9

